Neuronal Network Models for Generating Diverse Motion Patterns (多様な運動パターンを生成可能な神経回路モデル)

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1 Introduction

The central pattern generator (CPG) is known as a spinal neural circuit for biological rhythmic movements such as locomotion (e.g., walking, flying, swimming) [1]. In biological studies, CPGs have been known for their property of exhibiting stable oscillatory activities by receiving stationary inputs or tonic drives descending from a part of the brainstem called the mesencephalic locomotor region (MLR) [2]. The neural network architecture of CPG is suggested to be represented by a half-center, which is composed of reciprocal inhibitions between neurons [3].

To describe the activities of CPG, dynamical system models have been generally accepted. There are mainly three types of models: spiking neuron model, firing rate model, and phase oscillator model. In terms of mathematical tractability, spiking neuron models are hard to utilize for they consist of numerous parameters and variables. In addition, we are also interested in neural activities other than neural oscillations. The phase oscillator model does not fit this purpose because it is too abstract and specialized to describe interactive oscillators. In this study, we focus on one of the firing rate model called the Matsuoka oscillator model [4, 5, 6], which has been widely used as a CPG model for simulating biological movements and studying robot control of walking.

2 Model

We attempt to analyze a two-neuron case of the Matsuoka oscillator model, as the most simple and fundamental component of neural oscillations. The model is given by the following differential equations:

$$\tau_x \frac{dx_1}{dt} = -x_1 - by_1 - a_{12}z_2 + s_1, \tag{1a}$$

$$\tau_y \frac{dy_1}{dt} = -y_1 + z_1, \tag{1b}$$

$$\tau_x \frac{dx_2}{dt} = -x_2 - by_2 - a_{21}z_1 + rs_1, \tag{1c}$$

$$\tau_y \frac{dy_2}{dt} = -y_2 + z_2, \tag{1d}$$

$$z_1 = \max(x_1, 0), \tag{1e}$$

$$z_2 = \max(x_2, 0), \tag{1f}$$

where x_i is the (so-called) membrane potential or inner state of the i-th neuron ($i = 1, 2$), y_i is the variable of adaptation or fatigue, z_i is the firing rate, s_i is the constant input stimulus into the i-th neuron, $a_{ij} \geq 0$ is the synaptic weight from the i-th to j-th neuron ($j = 1, 2; j \neq i$), $b > 0$ is the constant determining adaptation intensity, and $\tau_x > 0$, $\tau_y > 0$ are the time constants of x_i, y_i, respectively.

For symplicity, we suppose

- symmetric synaptic connection $a = a_{12} = a_{21}$, and
- fixed value of s_1 whereas $s_2 = rs_1$.

We attempt to observe bifurcations between oscillations and steady states through changing the two parameters a and $r = s_2/s_1$, while the other parameters b, τ_x, τ_y, and s_1 are all fixed.

3 Previous Results and Questions

In the original papers [4, 5], a existence condition of oscillatory solutions of the model is derived concerning the parameters a and r:

$$a > 1 + \frac{\tau_x}{\tau_y}, \tag{2a}$$

$$\frac{a}{1+b} < r < \frac{1+b}{a}. \tag{2b}$$

Figure 1(a) shows an oscillation pattern observed when the condition (2a), (2b) are satisfied. The condition (2a), (2b) is actually derived as equivalent to the condition where no stable steady state exists in the phase space. Regarding this previous result, however, the following two points have been remained uncertain;

I. Steady states like Fig. 1(b) outside the oscillation range (2a), (2b) are not systematically formulized.
II. Bifurcation types between oscillations and steady states are unclear.

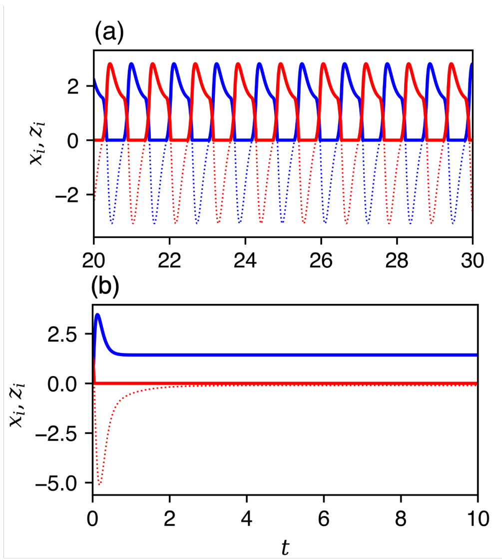

Fig. 1. Solutions of the model (1); x_i (dotted line), z_i (solid line), for $i = 1$ (blue) and $i = 2$ (red). (a) Oscillation. (b) Steady state.

4 Fixed Point

For the first question I, we performed fixed point analysis of the model (1). We divided the phase space $\{\boldsymbol{X} \in \mathbb{R}^4 \mid \boldsymbol{X} = [x_1 \quad y_1 \quad x_2 \quad y_2]\}$ into four domains A, B, C, and D, inside which different linear dynamics occurs. We introduced the concept "hidden" fixed point, and formulized the stabilty and existence of all fixed points $\boldsymbol{X}_A$, $\boldsymbol{X}_B$, $\boldsymbol{X}_C$, and $\boldsymbol{X}_D$ regarding the four linear dynamics, respectively. The result of this analysis is summarized in Fig. 2, which is the phase diagram of stable fixed points (steady states) and stable limit cycles (oscilaltions).

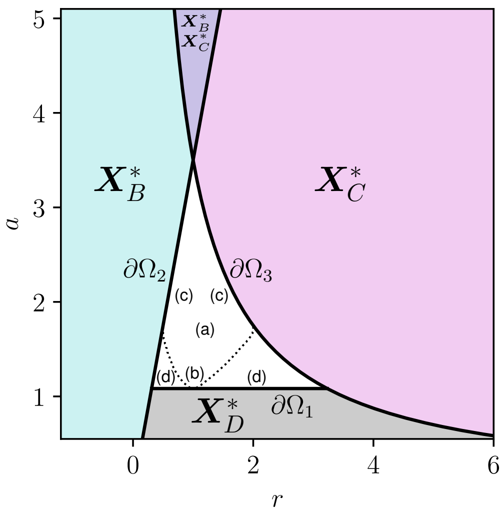

Fig. 2. Phase diagram of the stable fixed points $\boldsymbol{X}_B$, $\boldsymbol{X}_C$, $\boldsymbol{X}_D$, and stable limit cycles (a)–(d).

5 Bifurcation

5.1 Homoclinic-like bifurcation

According to the fixed point analysis, we can provide a basis to the second question II. At the boundaries between stable fixed points and stable limit cycles, there occurs a variety of bifurcations. We found one peculiar bifurcation type in this model, namely "homoclinic-like" bifurcation. This is observed at the borderline $\partial\Omega_2$ (or $\partial\Omega_3$) on Fig. 2. In this bifurcation, the stable fixed point $\boldsymbol{X}_B^*$ ($\boldsymbol{X}_C^*$) which is hidden in oscillatory states turns into existing, so that the system state succeeds in converging to it and the oscillation orbits disappear.

5.2 New approximation of oscillation period

An additional result concerning oscillation period was also obtained by the fixed point analysis. Figure 3 plots the numerical results of oscillation period T (circle) with respect to various values of a and fixed value $r = 1$. We proposed a new approximation curve (solid line) for the oscillation period T, which is given by

$$T_{\text{new}} = 2\tau_y \left[\ln \frac{1}{\delta} - \ln(1 + b - a) \right]. \quad (3)$$

This approximation is derived by evaluating the time spent to approach the hidden fixed points $\boldsymbol{X}_B^*$ and $\boldsymbol{X}_C^*$ in the linear dynamics. Unlike the previous approximation curve (dotted line) proposed by [6], the new curve (3) is well-fitted when a is larger and can express the logarithmic divergence of oscillation period in the supreme of a, where the homoclinic-like bifurcation occurs.

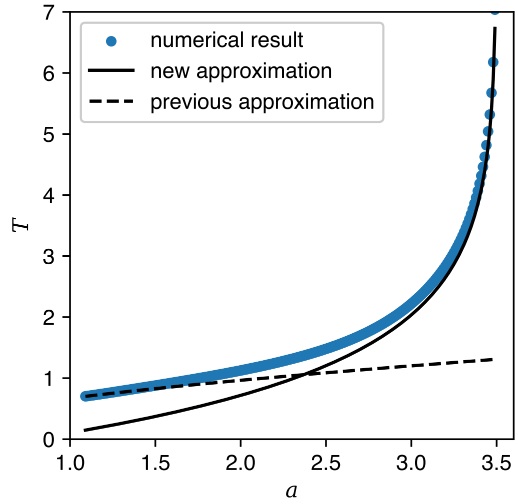

Fig. 3. Plot of the oscillation period T vs the symmetric synaptic weight a with fixed value $r = 1$

6 Conclusion

We analyzed the Matsuoka oscillator model as a representative model of the spinal neural circuit called CPG. Through analysis, we succeeded in formulizing the stability and existence of fixed points corresponding to steady states in the model. This formulization was followed by depicting the bifurcations between oscillations and steady states, and proposing a new approximation of oscillation period. The results about bifurcations would lead to predictions of noise-induced phenomena in the model and applications to biology or robot control studies regarding not only rhythmic movements but also transient movements [1].

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