Flow and transportation efficiency of an oscillator chain in Stokes flow

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In this research, a one-dimensional non-autonomous bead model with size d immersed in low Reynolds number fluid is considered, as a motor to a test particle with distance l away from the first bead (Fig. 1(a)). The dynamical equations are as the following:

$$\gamma\big(\dot{x}_i - v(x_i; x_j)\big) = F_i, \tag{1}$$

where γ is the drag coefficient, x_i is the position of bead $(i = 1, 2)$, $v(x_i; x_j)$ is the flow field at x_i obeying the Oseen flow, and

$$F_i = k(x_i^* + L(\phi_i(t)) - x_i) \tag{2}$$

is the force applied to i-th bead with k being the spring constant, x_i^* as the initial position of bead i. $L(\phi_i(t))$ is a sinusoidal function of phase functions $\phi_1 = \omega t$ and $\phi_2 = \omega t + \psi$ where ω is the angular frequency, so the difference is only the initial phase. The displacement of test particle R only depends on the net flow generated by beads and is derived perturbatively, where the dominant leading term is in the second order of perturbation parameter because the first order term vanishes over one cycle. This is shown in Fig. 1(b) along with the simulation result. The corresponding energy cost for the test particle transportation is calculated as well, and so we define the efficiency of the system representing the ability of transporting the test particle rightward.

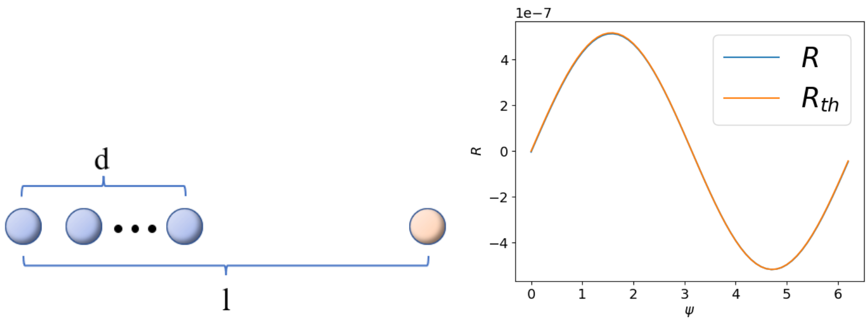

Figure 1: **Left**: The setup of model. Blue spheres are the motors and orange sphere is the test particle. First blue sphere is centered at the origin.
Right: The comparison between simulated and theoretical result. The simulation result is in blue, while the theoretical one is in orange. The theoretical result represents the leading-term perturbative result.

Moreover, the setup is extended by simulation to cases of multiple beads, while the distance between first and last bead is fixed, with their efficiencies discussed. The results shows an increment in efficiency from the coupling effect of cases with bead number more than 2, comparing to the partial 2-bead case. However, the increment saturates at the bead number of 6.

Another setup which is more practically applicable is considered as the model close to a wall. The image effect from wall arises, so the coupling effect is further enhanced comparing to cases without the wall effect.

An autonomous version of the setup is discussed as well. For a minimal 2-bead case with a simple sinusoidal phase sensitivity function, two fixed points are found which is very slightly dependent on the initial phase of phase sensitivity function. Also if the coupling strength is increased, the phases are now no longer converge and an oscillatory pattern is found.

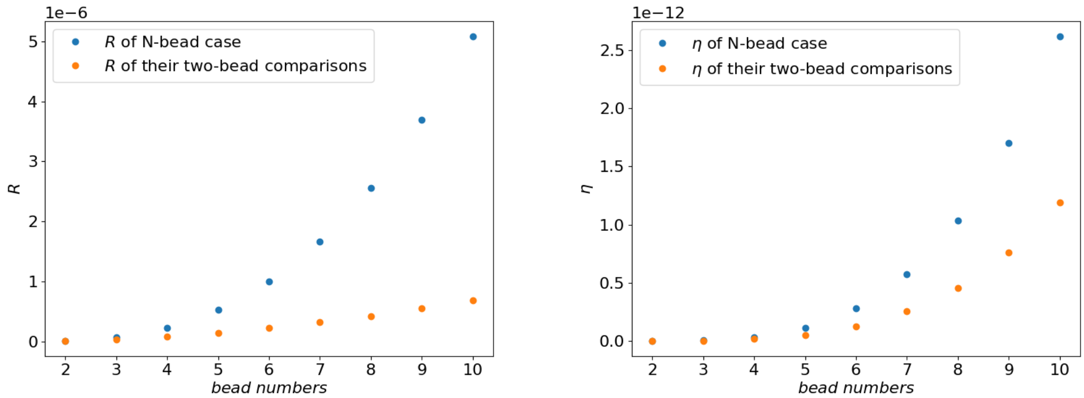

Figure 2: **Left**: Maximal R with respect to bead number N and the corresponding 2-bead analogs.
Right: Maximal efficiency of multiple beads cases and the corresponding 2-bead analogs.

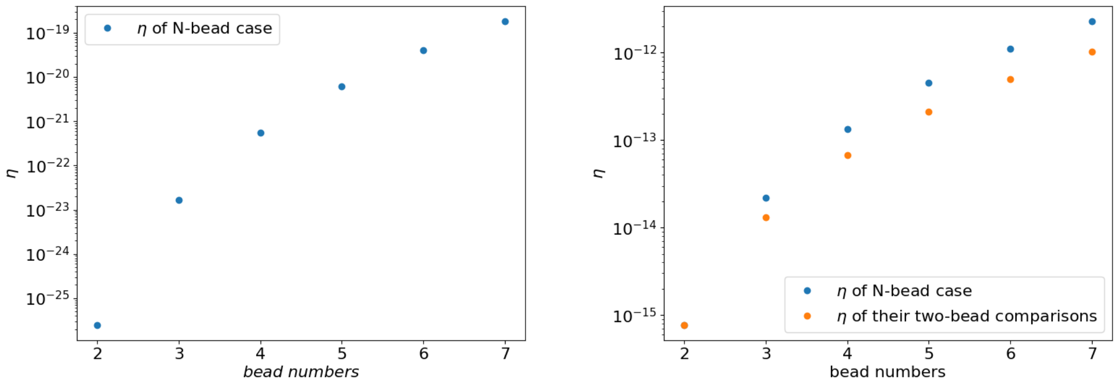

Figure 3: **Left**: Logarithmic plots of maximal η's with respect to bead number N, for cases with wall effect.
Right: the case without Oseen tensor for reference.
Stronger coupling effect is detected because the increments in scale is larger for the case with wall effect.

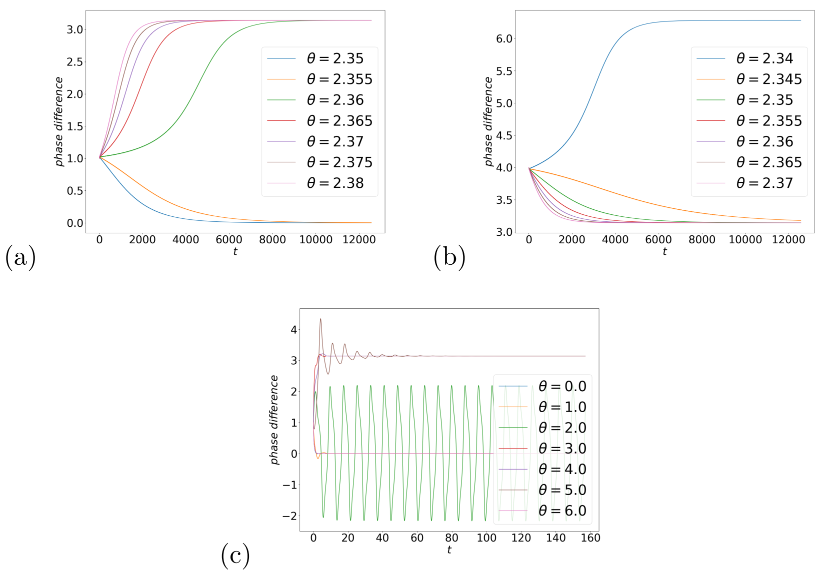

Figure 4: Plots of phase difference with respect to time. (a) The initial phase difference is 1.0. (b) The initial phase difference is 4.0. (c) The initial phase difference is 1.0, but the coupling strength is 500.0, comparing to 10.0 for (a) and (b).