Master Course Entrance Examination Problem Booklet

Information Physics and Computing

Tuesday, August 21, 2018 10:00~13:00

Answer three out of Problems 1-5.

Note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to fill the examinee's number and the problem number in the designated place at the top of each answer sheet. Do never put your name.

(5) Do not separate the draft papers from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that a problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with the problem numbers you selected.

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Problem 1

When we measure a distance by emitting a known sound signal $s(t)$ (t denotes the time) and detecting a time of flight, τ, of the sound, it is crucial to estimate a time delay of $s(t)$ in the observed signal at the receiver. Noise $n(t)$ that is uncorrelated with $s(t)$ exists in the transmission channel, and $s(t-\tau)+n(t)$ is observed at the receiver. We obtain an output signal by applying a filter that has the impulse response of $h(t)$ to the observed signal (see Fig. 1). $S(\omega)$ represents the Fourier transform of $s(t)$, $P_n(\omega)$ represents the power spectrum of $n(t)$, and $H(\omega)$ represents the Fourier transform of $h(t)$, where ω denotes the angular frequency. Hereafter, we define the following signal-to-noise ratio criterion $J(t)$:

$$J(t) = \frac{|h(t) * s(t-\tau)|^2}{\int_{-\infty}^{\infty} |h(t) * n(t)|^2 \mathrm{d}t},$$

where $*$ denotes the convolution operator. We derive an impulse response $h(t)$ of the optimal filter that can maximize $J(t)$ at $t=\tau$. Answer the following questions.

(1) Prove that, in general, the following equation holds in terms of the convolution operation between arbitrary $x(t)$ and $y(t)$:

$$\int_{-\infty}^{\infty} x(t) * y(t) \exp(-\mathrm{j}\omega t)\mathrm{d}t = X(\omega)Y(\omega), \tag{1}$$

where $X(\omega)$ and $Y(\omega)$ are the Fourier transforms of $x(t)$ and $y(t)$, respectively, and j is the imaginary unit.

(2) First, we consider the numerator of $J(t)$. Show the filtered output of $s(t-\tau)$, i.e., $h(t) * s(t-\tau)$, by using $S(\omega)$ and $H(\omega)$ (the form of integration can be acceptable), where you can use the relation of Eq. (1). Next, we consider the denominator of $J(t)$. Show $\int_{-\infty}^{\infty} |h(t) * n(t)|^2 \mathrm{d}t$ by using $P_n(\omega)$ and $H(\omega)$ (the form of integration can be acceptable).

(3) On the basis of the results in Question (2), prove the following inequality

$$|h(t) * s(t-\tau)|^2 \leq \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{P_n(\omega)} \mathrm{d}\omega \cdot \int_{-\infty}^{\infty} |h(t) * n(t)|^2 \mathrm{d}t.$$

(4) On the basis of the result in Question (3), show the optimal filter's frequency characteristics $H(\omega)$ by using $S(\omega)$ and $P_n(\omega)$. Also, if the noise $n(t)$ is white, i.e., $P_n(\omega) = N$; N is a constant value, find the impulse response $h(t)$ of the optimal filter. Furthermore, under the above-mentioned condition, we consider

the case that the frequency characteristics $S(\omega)$ of the emitted signal are given by

$$S(\omega) = \begin{cases} \sqrt{1 - \frac{|\omega|}{\omega_0}} \exp(\mathrm{j}\phi(\omega)) & (|\omega| \leq \omega_0) \\ 0 \quad \text{(otherwise)} \end{cases},$$

where ω_0 is a bandwidth of the signal and $\phi(\omega)$ represents a phase function that is an arbitrary odd function. Calculate the waveform of $h(t) * s(t - \tau)$ and illustrate it in the optimal filter's output.

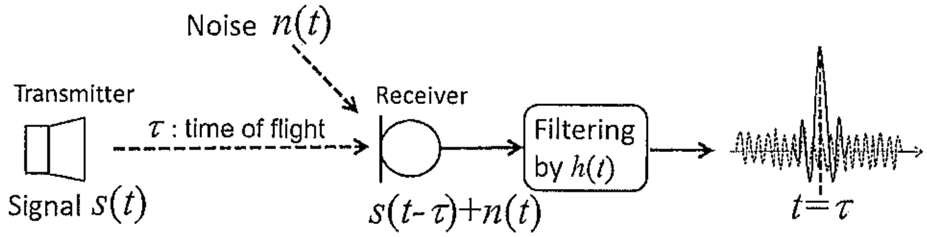

Fig. 1 Overview of the measurement system for time of flight

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Problem 2

Figure 1 shows a two-port network in which the voltage and current of the input terminal are V_1 and I_1, respectively, and those of the output terminal are V_2 and I_2, respectively. The input and output relation of the network is represented by cascade matrix $\boldsymbol{F}$ as follows. Note that I_2=0 and V_2=0 indicate open and short circuit of the output terminal, respectively.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \boldsymbol{F} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \quad F = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}, \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Answer the following questions.

(1) When the two-port networks represented by $\boldsymbol{F}_{\mathrm{L}}$ and $\boldsymbol{F}_{\mathrm{R}}$ are connected in cascade as shown in Fig. 2, express the cascade matrix of the synthesized two-port network, $\boldsymbol{F}_{\mathrm{LR}}$, in terms of $\boldsymbol{F}_{\mathrm{L}}$ and $\boldsymbol{F}_{\mathrm{R}}$.

(2) Find cascade matrices $\boldsymbol{F}_{\mathrm{V}}$ and $\boldsymbol{F}_{\mathrm{H}}$ representing the two two-port networks as shown in Fig. 3.

(3) If the two two-port networks are identical to each other as shown in Fig. 4, express Z_{AB}, Z_{BC}, and Z_{CA} by using Z_A, Z_B, and Z_C.

(4) Suppose that we have one operational amplifier having the ideal characteristics, two resistors of resistance r, and two resistors of resistance $10r$. By using them, we want to build an inverting amplifier circuit of which gain is as large as possible. Illustrate its circuit diagram, explain the reason that it works as an inverting amplifier, and find its gain.

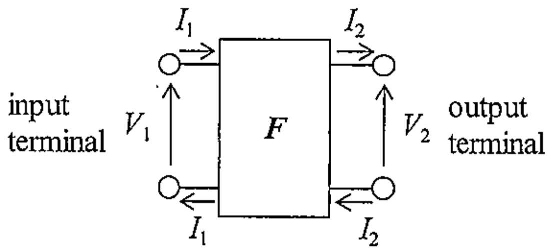

Fig. 1

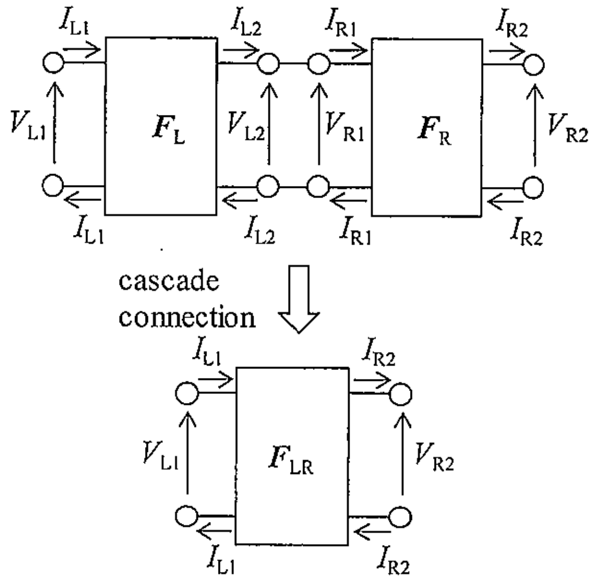

Fig. 2

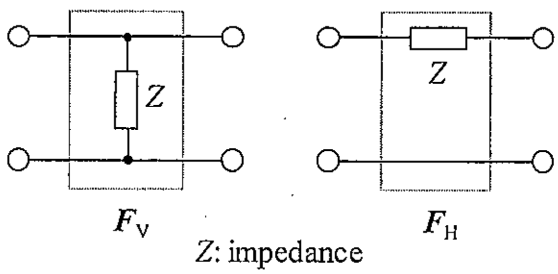

Fig. 3

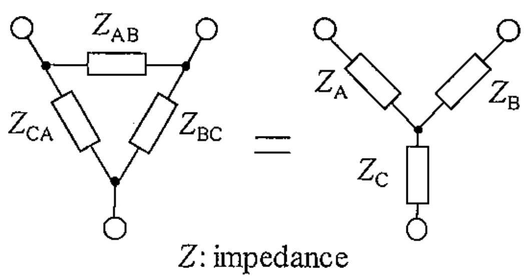

Fig. 4

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Problem 3

Consider system (1) given by a state space representation;

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = Ax(t) + Bu(t). \tag{1}$$

Answer the following questions.

(1) Assume that system matrices A and B are given by

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{2}$$

Show that system (1) is controllable.

(2) Show the solution $x(t)$ of system (1) by using A, B, $x(0)$, and $u(t)$.

(3) Assume that system (1) is controllable. Consider to design a control input $u(t)$, $0 \leq t \leq L$ $(L > 0)$ for a given initial condition $x(0) = x_o$ and a given objective state vector x_L in order to satisfy $x(L) = x_L$ at time $t = L$. One of such $u(t)$ is given by a form

$$u(t) = B^\top \exp(-A^\top t)\xi, \; 0 \leq t \leq L.$$

Show vector ξ. Next assume that system (1) is not controllable. Explain the possible x_o and x_L.

(4) (a) Assume that system matrices A and B are given by Eq. (2). In the case of $x_o = O$, show $u(t)$, $0 \leq t \leq L$ by using x_{11} and x_{12} such that

$$x(L) = x_L := \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}, \; L = 1.$$

(b) In Question (4)–(a) for a certain $x(1) = x_1$, let $u(t) = 0$, $1 < t$, then $x(t)$ satisfies $x(t) = x_1$, $1 \leq t$. Show x_1.

(c) In Question (4)–(a), let L be a smaller number compared with 1. Explain the behavior of $u(t)$, $0 \leq t \leq L$ in the control input design.

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Problem 4

Consider a processor using 32-bit address space and byte addressing. Answer the following questions about caches of this processor. Note that 1 KiB means 2^{10} bytes.

(1) In a direct mapped cache with 128 16-byte cache lines, answer the cache line number where the 1-byte data at the address 9540 (in decimal) in memory are stored. Assume that the first cache line number is 0.

(2) Find the tag memory capacity of each of the following caches.

   (a) A direct mapped cache whose cache line size is 16 bytes and data capacity is 64 KiB.

   (b) A 4-way set associative cache whose cache line size is 32 bytes and data capacity is 512 KiB.

(3) In an n-way set associative cache with the LRU replacement algorithm, describe two demerits of increasing n when the capacity and the line size of the cache are fixed.

(4) Consider an n-way set associative data cache whose replacement algorithm is LRU, line size is 32 bytes, and data capacity is 64 KiB. Assume that the processor's operating frequency is 2 GHz and the cache miss penalty is 100 ns.

   (a) The program of Fig. 1 written in the assembly language of Fig. 2 was executed on this processor from "Start" to "End". Assume that the cycles per instruction was one when no data cache miss occurred. Find the total number of executed instructions, and the cycles per instruction considering data cache misses for each case of n=1 and n=2. Labels themselves are not included in the number of instructions.

   (b) In the case of n=1, find the cycles per instruction considering data cache misses when the address of "Array2" in the program of Fig. 1 is changed to 131056 (in decimal).

   (c) In the case of n=1, write a program that performs the calculation equivalent to the program of Fig. 1 in the assembly language of Fig. 2 so that the number of data cache misses is reduced as much as possible. Also, find the number of data cache misses in this program. Here, do not change the address of "Array1" or "Array2", or values stored in the memory at that address.

```
Start:
    set $r0, 0
    set $r1, 0
    set $r15, 8
Loop:
    load $r2, Array1[$r1]
    load $r3, Array2[$r1]
    mul $r2, $r3
    add $r0, $r2
    add $r1, 1
    sub $r15, 1
    jnz $r15, Loop
End:
    halt

.Address 65536
Array1: dw 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
.Address 131072
Array2: dw 4, 9, 3, 8, 5, 12, 1, 6, 11, 2, 10, 7
```

Fig. 1

(In the following, R represents a register, IR represents an immediate value or a register, and A represents an address)

set R, IR	set IR to R
load R, A	load one-word data at A to R
add R, IR	add IR to R, and store the result to R
sub R, IR	subtract IR from R, and store the result to R
mul R, IR	multiply R by IR, and store the result to R
jnz R, A	If R is not zero, then jump to A. Otherwise, go to the next instruction
halt	stop the execution

- There are 16 registers ($r0 - $r15), and each of them is one word.
- A string followed by a colon is a label and can be used as an address.
- An address is expressed either by a label or in the form of "label[register]." In the latter case, the address is the sum of the register value multiplied by four and the label value.
- ".Address *value*" is a directive to the assembler, indicating that the address of the label in the next line is set to the *value* (in decimal).
- "*label*: dw" indicates that the following values separated by commas are stored in memory starting from the address of the *label* as one-word (4 bytes) data each in this order.

Fig. 2

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Problem 5

Consider a pendulum which consists of two elongated rigid bodies R_1 and R_2, each having the same mass m, the same length h, and the same moment of inertia I about the center of gravity, as shown in Fig. 1. A ceiling and one end of rigid body R_1 are connected by rotational joint A_1, and the other end of rigid body R_1 and one end of rigid body R_2 are connected by rotational joint A_2. For $i = 1$ and 2, let h_i be the distance between the center of joint A_i and the center of gravity of rigid body R_i, and θ_i be the angle formed by the vertical line and rigid body R_i. Assume that joint friction and air resistance can be ignored, and the pendulum moves in the two-dimensional vertical plane. Denote the gravitational acceleration by g. Answer the following questions.

(1) Suppose that the rotational motion of joint A_2 is fixed so that two rigid bodies R_1 and R_2 always make an angle of 0, which is equivalent to $\theta_2 = \theta_1$. Find the moment of inertia of the pendulum, J, about joint A_1.

(2) The pendulum in Question (1) is raised to the angle of $\theta_1 = \gamma$ and then is released from rest. Find the maximum speed at the head (free end) of the pendulum. If necessary, use J.

A rotational motor is attached to each joint for generating torque. Assume that the mass and moment of inertia of motors can be ignored.

(3) (a) The pendulum is at rest at angles of $\theta_1 = \alpha_1$ and $\theta_2 = \alpha_2$. Find the torque generated by each motor.

(b) The attitude of the pendulum is changed from rest in Question (3)-(a) to the angle of $\theta_2 = \beta_2$ while keeping the angle of $\theta_1 = \alpha_1$, and then is changed to the angle of $\theta_1 = \beta_1$ while keeping the angle of $\theta_2 = \beta_2$, as shown in Fig. 2. Find the work done by motors and the change in potential energy of the pendulum in the overall path, and show that they coincide with each other. Here, assume that the movement of the pendulum is a quasi-static process.

(4) The pendulum moves according to the formulas of $\dfrac{\mathrm{d}^2\theta_1}{\mathrm{d}t^2} = 0$ and $\dfrac{\mathrm{d}^2\theta_2}{\mathrm{d}t^2} = 0$.

Find the torque generated by each motor as a function of $\theta_1, \theta_2, \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t}$, and $\dfrac{\mathrm{d}\theta_2}{\mathrm{d}t}$.

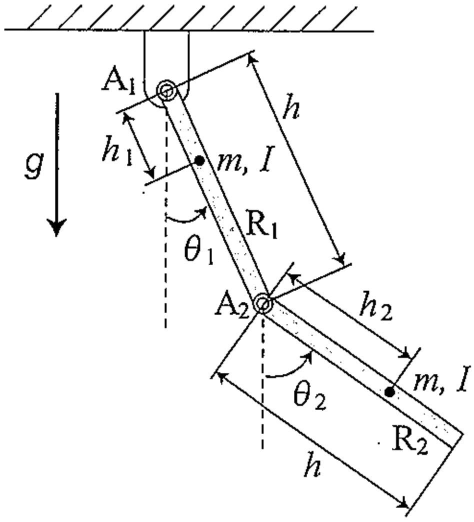

Fig. 1

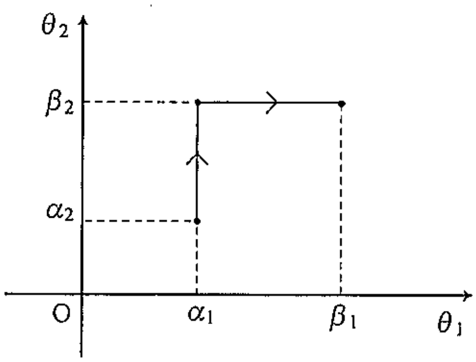

Fig. 2

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