Specialized Subjects

10:00-12:30, Monday, August 19, 2024

Instructions

1. Do not open this booklet before the examination begins.

2. This booklet contains five problems. The number of pages is six excluding this cover sheet and blank pages. If you find missing or badly printed pages, ask the proctor for exchange.

3. Answer three problems. You can select any three out of the five. Your answer to each problem should be written on a separate sheet. You may use the reverse side of the sheet if necessary.

4. Fill the top parts of your three answer sheets as instructed below. Before submitting your answer sheets, make sure that the top parts are correctly filled.

専　門　科　目

第　　問

↑ Write the problem No.

受験番号				

↑ Write the examinee No.

5. Submit all the three answer sheets with the examinee number and the problem number, even if your answer is blank.

6. Answer either in Japanese or in English.

7. This booklet and the scratch paper must be returned at the end of the examination.

8. This English translation is supplemental and provided for the convenience of applicants. The Japanese version is the formal one.

Problem 1

Consider the circuit consisting of a power source E, resistances R_1 and R_2, an inductance L_1, a capacitance C_2, and an ideal transformer with a turns ratio of $n : 1$ (where n is a positive natural number), as shown in the figure. Let s and t be the Laplace variable and the time, respectively. Answer the following questions.

(1) Suppose that the voltage $v_0(t)$ of the power source E is represented as a sine-wave with the amplitude ϵ and the angular frequency ω as $v_0(t) = \epsilon \sin(\omega t)$. Express the secondary voltage $V_2(s)$ of the transformer using the primary voltage $V_1(s)$.

(2) Under the condition of (1), derive the impedance $Z_2(s)$ on the secondary side of the transformer.

(3) Under the condition of (1), derive the impedance $Z_0(s)$ of the entire circuit as seen from the power source.

(4) Under the condition of (1), express the secondary voltage $V_2(s)$ using the Laplace transform of the power source voltage $V_0(s)$.

(5) Consider the case where the voltage $v_0(t)$ of the power source E is represented as a unit step voltage $v_0(t) = u(t)$. When $R_1 = R_2 = 1$ [Ω], $L_1 = 1$ [H], and $C_2 = 1$ [F], show the time variation of the voltage $v_2(t)$ on the secondary side of the transformer.

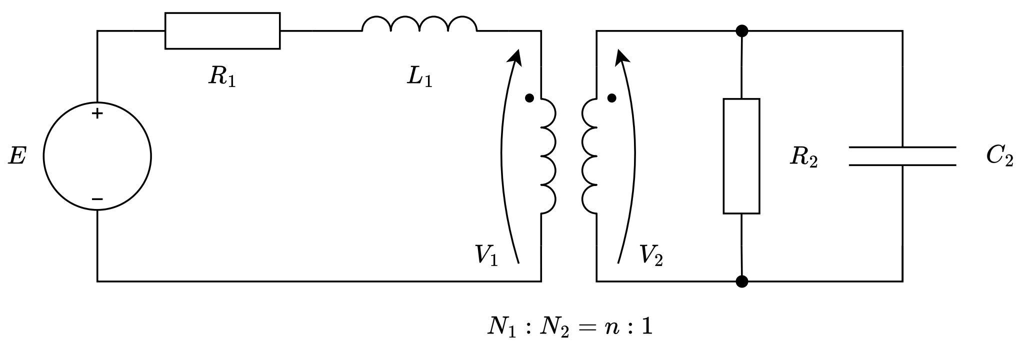

Fig.

Problem 2

Consider the synchronous sequential circuit shown in Fig. 1. Here, D_0, D_1, and D_2 represent the input signals for each D Flip Flop (DFF), and Q_0, Q_1, and Q_2 represent the coresponding output signals of each DFF. The input x takes a value of either 0 or 1. Answer the following questions.

(1) Represent D_0, D_1, and D_2 as logical expressions using the signals from Q_0, Q_1, Q_2, and the input x as necessary.

(2) Define the states of the circuit in Fig. 1 appropriately and create a state transition table. Consider only the case when the input x is 1.

(3) Create a state transition diagram of the circuit in Fig. 1 in the form of a Mealy graph. Consider only the case when the input x is 0.

(4) Describe how the output z of the circuit in Fig. 1 changes with the clock. Assume the input x is 1, and each DFF in the circuit is initialized so that its output is 0.

(5) Design a synchronous sequential circuit that outputs 1, 1, 1, 0, 1, 0, 0 in this order, and draw the circuit using the MIL symbols (Fig. 2). The types of elements that can be used in the circuit are NOT, NAND, NOR, and DFF. This circuit has no input and repeats the output sequence of 1, 1, 1, 0, 1, 0, 0 with a period of 7 clock cycles as long as the clock continues. Additionally, assume each DFF in the circuit is initialized so that its output is 0.

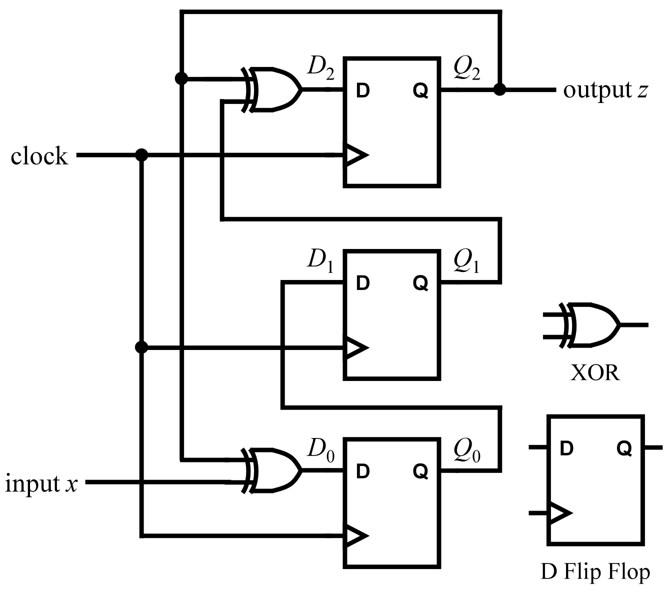

Fig. 1

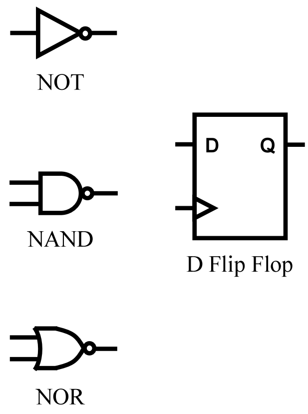

Fig. 2

Problem 3

The Minimum Spanning Tree (MST) problem is the problem of finding a subgraph that satisfies the following conditions, given an undirected graph $G = (V, E)$ with a vertex set V, an edge set E, and each edge having a weight:

- It includes all vertices V of the graph G.
- It has a tree structure with no cycles.
- The total weight of the edges is minimized.

When answering, assume the following conditions:

- When using a sorting function, assume the time complexity is $O(n \log n)$ for an array of length n.
- Determining which set each vertex belongs to, as well as the union operation of two different sets, can be performed in constant time, for example, using the disjoint-set data structure (Union-Find).
- When using a priority queue, assume it is implemented using a heap.

Answer the following questions.

(1) Briefly present the approach and pseudocode of an algorithm to solve the Minimum Spanning Tree problem. The algorithm should be deterministic and have a time complexity of $O(|E| \log |V|)$ where $|E|$ is the number of elements in the set E and $|V|$ is the number of elements in the set V.

(2) Using the algorithm presented in (1), show the Minimum Spanning Tree and the total weight of the edges for the graph in Figure.

(3) Briefly present the approach and pseudocode of an algorithm to find the spanning tree with the second smallest total edge weight (hereafter referred to as the Second MST). Also, show the time complexity.

(4) Using the algorithm presented in (3), show the Second MST and the total weight of the edges for the graph in Figure.

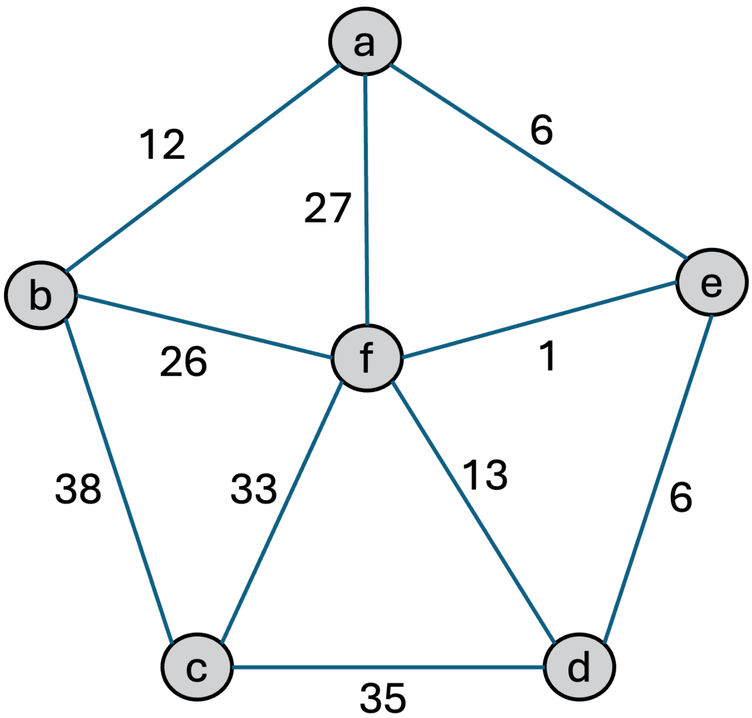

Fig.

Problem 4

Consider the problem of learning a model that takes a D-dimensional vector as an input and outputs a scalar value. Below, $\top$ denotes the transpose of a vector or a matrix, $\mathbb{R}$ represents the set of all real numbers, $\mathbb{R}^D$ represents the set of all D-dimensional real column vectors, and $\mathbb{R}^{D\times D}$ represents the set of all real square matrices of order D. For training data, N D-dimensional input vectors $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N$ and their corresponding outputs $y_1, \ldots, y_N$ are given. Here, $\boldsymbol{x}_i \in \mathbb{R}^D$ and $y_i \in \mathbb{R}$ $(1 \leq i \leq N)$. The matrix consisting of each input vector arranged vertically as rows is written as $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N]^\top$. Furthermore, the outputs are written together as $\boldsymbol{y} = [y_1, \ldots, y_N]^\top$. Based on these, the D-dimensional parameter $\boldsymbol{\beta} \in \mathbb{R}^D$ is learned. Given any input data $\boldsymbol{x} \in \mathbb{R}^D$, the output is estimated as $\hat{y} = \boldsymbol{\beta}^\top \boldsymbol{x}$. Answer the following questions.

(1) For the i-th training input $\boldsymbol{x}_i$, let the estimated output using the learned $\boldsymbol{\beta}$ be $\hat{y}_i = \boldsymbol{\beta}^\top \boldsymbol{x}_i$ $(1 \leq i \leq N)$. Let the error between the true output y_i and the estimated output $\hat{y}_i$ be $e_i = y_i - \hat{y}_i$. In this case, express the sum of squared errors $E = \sum_{i=1}^N e_i^2$ using $\boldsymbol{\beta}$, $\boldsymbol{X}$, and $\boldsymbol{y}$.

(2) Find the necessary and sufficient condition for the parameter $\boldsymbol{\beta}$ that minimizes E to be uniquely determined. Also, obtain $\boldsymbol{\beta}$ in this case. Here, for a variable $\boldsymbol{a} \in \mathbb{R}^D$, a constant $\boldsymbol{b} \in \mathbb{R}^D$, and a constant $\boldsymbol{C} \in \mathbb{R}^{D\times D}$, you may use the following relations:

$$\frac{\partial}{\partial \boldsymbol{a}}(\boldsymbol{b}^\top \boldsymbol{a}) = \boldsymbol{b},$$

$$\frac{\partial}{\partial \boldsymbol{a}}(\boldsymbol{a}^\top \boldsymbol{C} \boldsymbol{a}) = (\boldsymbol{C} + \boldsymbol{C}^\top)\boldsymbol{a}.$$

(3) If the parameter $\boldsymbol{\beta}$ that minimizes E is not uniquely determined, describe qualitatively what properties the input data $\boldsymbol{X}$ has.

(4) Suppose that the importance $w_i > 0$ is given to the i-th training data $(1 \leq i \leq N)$. Let $E_w = \sum_{i=1}^N w_i e_i^2$ be the sum of squared errors considering this importance. Find the necessary and sufficient condition for the parameter $\boldsymbol{\beta}$ that minimizes E to be uniquely determined. Also, obtain $\boldsymbol{\beta}$ in this case. You may define new variable(s) if necessary.

(5) Find $\boldsymbol{\beta}$ that minimizes E_w when the variables take the following values.

$$\begin{aligned}
\boldsymbol{x}_1 &= [1, 0, 1]^\top, \quad y_1 = 2, \quad w_1 = 1, \\
\boldsymbol{x}_2 &= [0, 1, 1]^\top, \quad y_2 = 3, \quad w_2 = 1, \\
\boldsymbol{x}_3 &= [2, 0, 1]^\top, \quad y_3 = 3, \quad w_3 = 2, \\
\boldsymbol{x}_4 &= [1, 1, 0]^\top, \quad y_4 = 1, \quad w_4 = 1.
\end{aligned}$$

Problem 5

Answer the following questions about discrete-time signal processing.

(1) Describe briefly the functionality and role of an anti-aliasing filter.

(2) For a linear time-invariant and causal discrete-time signal processing system L, let the transfer function be

$$H(z) = \frac{\alpha z^2}{\alpha\beta z^2 - \alpha\beta z + \beta - \gamma},$$

where z is a complex variable, α, β, γ are real constant numbers, and $\alpha, \beta > 0$. In this case, find the difference equation for the relationship between the input $x[n]$ and the output $y[n]$ of the discrete-time signal for L, where n is an integer.

(3) In (2), find the range of γ such that L is stable when $\alpha = 4$ and $\beta = 1$.

(4) In (2), when $\alpha = 4$ and $\beta = \gamma = \frac{1}{2}$, find the magnitude response $|H(e^{j\Omega})|$ and the phase response $\angle H(e^{j\Omega})$ in L, where j is the imaginary unit and Ω is the angular frequency.

(5) In (2), when $\alpha = 4, \beta = 1$, and $\gamma = \frac{1}{9}$, find the impulse response $h[n]$ of L using the following discrete-time unit step signal $u[n]$.

$$u[n] = \begin{cases} 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$