Specialized Subjects

10:00-12:30, Monday, August 23, 2021

Instructions

1. This booklet contains five problems. The number of pages is six excluding this cover sheet and blank pages.

2. Answer three problems. You can select any three out of the five. Your answer to each problem should be written on a separate sheet. Answers to a single problem may be spread over several answer sheets. However, answers to multiple problems must not be written on one answer sheet.

3. Write the problem number that you have chosen and your examinee number on top of each answer sheet.

4. Submit the answer sheets for three problems. Even if you have solved only one or two problems, you must submit answer sheets with the examinee number and the problem number.

5. Answer either in Japanese or in English.

6. This booklet cannot be downloaded nor saved to your local computers.

7. This English translation is supplemental and provided for convenience of applicants. The Japanese version is the formal one.

Problem 1

Consider the circuit as shown in Fig. 1, which consists of a power supply, an inductor (inductance L), and a resistor (resistance R). Let t, $e(t)$, and $i(t)$ be the time, the power supply voltage, and the value of the current, respectively. Note that the direction of the current is as shown in the figure. Answer the following questions. If necessary, refer to the Laplace transform table shown in Table. Note that the step function $u(t)$ is defined as follows:

$$u(t) = \begin{cases} 1 & 0 \leq t, \\ 0 & \text{otherwise.} \end{cases}$$

(1) Suppose that the supply voltage is a constant E (i.e., $e(t) = E$), and we close the switch at time $t = 0$. Let $I(s)$ be the Laplace transform of the current $i(t)$. Find $I(s)$ by constructing the circuit equation and applying the Laplace transform to it. Also, find $i(t)$ by the inverse Laplace transform of $I(s)$.

(2) Consider the function $v(t)$ of t shown in Fig. 2. Find $V(s)$ by applying the Laplace tranfsorm to $v(t)$.

(3) Suppose that the supply voltage of the circuit in Fig. 1 is changed from a constant voltage to the one in Fig. 2 (i.e., $e(t) = v(t)$). Suppose that we close the switch at $t = 0$. Also, assume that $E = R = L = T = 1$. Find $I(s)$ by applying the Laplace transform to the circuit equation. Also, find $i(t)$ by the Laplace inverse transform of $I(s)$.

(4) For $i(t)$ obtained in (3), illustrate its shape as a function of t.

Table

Time domain: $f(t), t \geq 0$	s domain: $F(s)$	Comment
$u(t)$	$\frac{1}{s}$	
$e^{\alpha t}$	$\frac{1}{s-\alpha}$	$\alpha \in \mathbb{R}$
t^n	$\frac{n!}{s^{n+1}}$	
$f(t-\alpha) \cdot u(t-\alpha)$	$e^{-s\alpha}F(s)$	Shift on the t axis. $\alpha > 0$

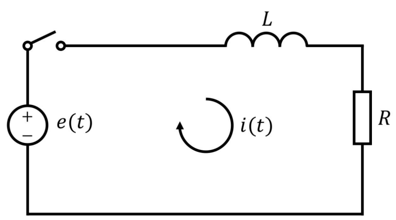

Fig. 1

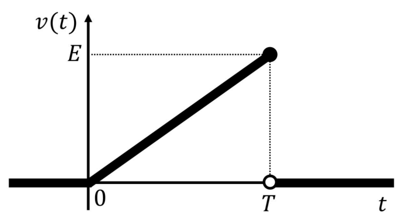

Fig. 2

Problem 2

Suppose a computer comprising memory hierarchy presented in Fig.

(1) Describe the characteristics of the cache and the characteristics of the auxiliary memory (secondary storage) in comparison with the main memory. Present a specific example of a memory device utilized for the cache and a specific example of a memory device utilized for the auxiliary memory.

(2) Suppose that the computer presented in Fig. achieved a CPI (Cycles Per Instruction) of 1.50 in executing a program A which induced no memory stalls, while the computer yielded an instruction cache miss rate of 1 % and a data cache miss rate of 4 % in executing another program B. In either case, no inputs or outputs to the auxiliary memory were observed, all the instructions were executed serially, and the branch prediction was not performed. Assume that 60 % of all the instructions of the program B were load-store instructions, and a penalty of 100 cycles was incurred for a cache miss. Find the CPI of the computer in executing the program B.

(3) Suppose that, in the case of (2), after secondary cache was installed between the cache and the main memory, the rate of accessing the main memory for the instruction fetch and the data load-store was reduced to 0.5 % in executing the program B. In this case, no inputs or outputs to the auxiliary memory were observed, all the instructions were executed serially, and the branch prediction was not performed. Assume that an access to the secondary cache incurred 10 cycles. Find the speedup ratio that the secondary cache achieved for the execution of the program B.

(4) A controller of virtual memory usually performs a write to the auxiliary memory in the write-back policy. Describe the adequacy of employing the write-back policy for the controller in terms of the execution speed of computers and the persistence of data.

(5) Suppose virtual memory where the virtual address length is 32 bits, the page size is 4096 bytes, and the page entry length is 4 bytes. Assume that the page table is structured with a single array. Find the size of the page table necessary for executing a program which invokes 100 concurrent processes.

(6) Briefly describe two effective approaches to reduce the size of the page table.

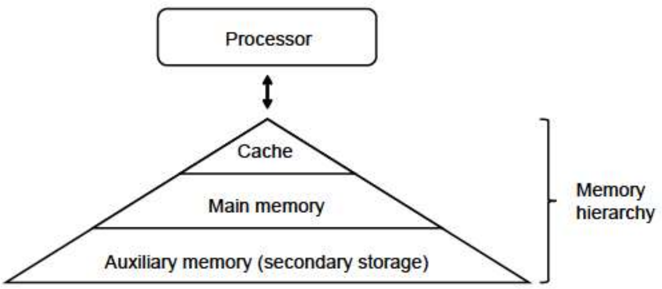

Fig.

Problem 3

Let T and P be two arrays of length n and m $(0 < m \leq n)$, respectively, and the elements of the arrays be non-negative integers less than k (≥ 2). Answer the following questions.

(1) Consider an algorithm Count_Pattern that counts T's subarrays, each of which is identical to P.

```
COUNT_PATTERN(T, P, n, m, k):
    c = 0
    for i = 0 to n − m do
        for j = 0 to m do
            ┌─────────┐
            │   (Q)   │
            └─────────┘
            if T[i + j] ≠ P[j] then
                break
    return c
```

Complete this pseudocode by filling in $\boxed{\text{(Q)}}$.

(2) Let us apply Count_Pattern to $T = \langle 1, 1, 1, 0, 1, 1, 0 \rangle$, $P = \langle 1, 1, 0 \rangle$, $n = 7$, and $m = 3$. Show the transition of values of i, j, and c at when evaluating $T[i + j] \neq P[j]$ in the inner loop during the execution of the algorithm.

(3) Suppose that Count_Pattern is applied to T and P which are randomly chosen for given n, m, and k. Let us denote by a random variable $X_{n,m,k}$ the number of times evaluating $T[i + j] \neq P[j]$ in the inner loop of Count_Pattern. Express $\mathbb{E}[X_{n,m,k}]$, the expected value of $X_{n,m,k}$, using n, m, and k. Also, prove $\mathbb{E}[X_{n,m,k}] < 2n$.

(4) By using a deterministic finite automaton (DFA) that is built in advance to match the input with P, we can count T's subarrays, each of which is identical to P, with the worst-case time complexity of $O(n)$. Depict a DFA to be built for $P = \langle 1, 1, 0 \rangle$ and $k = 3$.

(5) Assume a bijection f to a non-negative integer from an m-element array P' of non-negative integers less than k. Using this bijection f, we can count T's subarrays, each of which is identical to P, with the worst-case time complexity of $O(n)$. Show a pseudocode of Count_Pattern that is modified to run in $O(n)$ by implementing f as a function with P', m, and k as arguments. Here, implement f as a recursive function based on addition and multiplication to reduce the number of times evaluating the addition and multiplication as much as possible.

Problem 4

Consider a digital wireless communication system in which a transmitted signal is given by $d(t) = \mathrm{Re}\left[sg(t)e^{j2\pi ft}\right]$. Here, s is a complex-valued symbol in an equivalent low-pass representation, $g(t)$ is a pulse waveform, and f is a carrier center frequency. Assume that the occurrence probability of each symbol in a signal-space diagram is equal. Also, Re denotes the real part of a complex value. Answer the following questions.

(1) Describe two roles of a low-pass filter employed at a transceiver.

(2) Fig. 1 and Fig. 2 show signal-space diagrams for 8 phase-shift keying (8PSK) and 16 quadrature amplitude modulation (16QAM), respectively. Show the number of information bits per symbol for each modulation scheme.

(3) Consider that Gray coding is used for modulating information bits onto a complex-valued symbol. Show in signal-space diagrams mappings from information bits to complex-valued symbols for 8PSK and 16QAM, respectively. Note that Gray coding has the property that the Hamming distance between adjacent symbols is 1.

(4) Consider that the radius for 8PSK is A (Fig. 1). Under the condition that the average signal energy per information bit for 8PSK is the same as that for 16QAM, show the minimum symbol distance B for 16QAM using A.

(5) Under the conditions of (3) and (4), consider an additive white Gaussian noise channel with a power spectral density N_0. The probability that a transmitted symbol is incorrectly detected as another symbol can be approximated by $Q\left(x/\sqrt{2N_0}\right)$ using the Q function, when the distance between these two symbols in a signal-space diagram is x. Also, assume that misdetection occurs only between symbols with the minimum symbol distance. Show the average bit error rate of 16QAM using Q, A, and N_0.

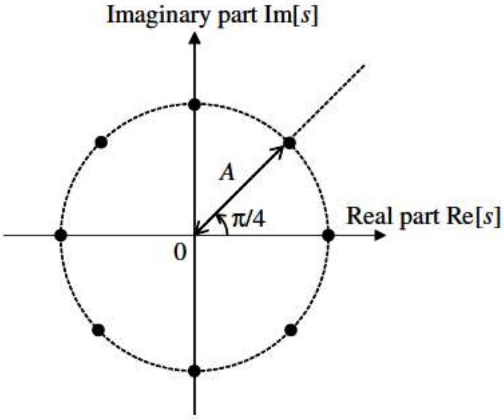

Fig. 1

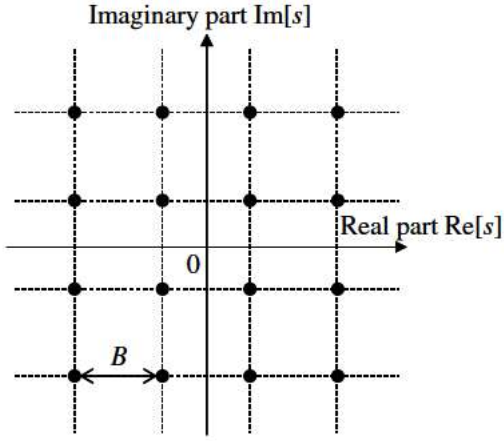

Fig. 2

Problem 5

We consider to obtain the output of an ideal low-pass filter when a single square-wave pulse is given as the input. Assume that the single square-wave pulse is represented as:

$$x(t) = \begin{cases} 1 & (\text{if } |t| \leq \frac{t_w}{2}) \\ 0 & (\text{if } |t| > \frac{t_w}{2}) \end{cases}$$

and the ideal low-pass filter is represented as:

$$H(\omega) = \begin{cases} e^{-j\omega t_d} & (\text{if } |\omega| \leq \omega_m) \\ 0 & (\text{if } |\omega| > \omega_m) \end{cases}$$

Assume that $t_w > 0$, $t_d > 0$, and $\omega_m > 0$. Answer the following questions.

You can use the following formulae as needed.
The Fourier transform of the signal $x(t)$:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt.$$

The inverse Fourier transform of $X(\omega)$:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega.$$

(1) Let $x_0(t)$ be the signal having no angular frequency component higher than ω_m (i.e., $|\omega| \leq \omega_m$), and assume that we input this to the ideal low-pass filter $H(\omega)$. Compute the output signal $y_0(t)$.

(2) Compute the Fourier transform $X(\omega)$ of the single square-wave pulse $x(t)$.

(3) Let $y(t)$ be the output signal when the single square-wave pulse $x(t)$ is given to the ideal low-pass filter $H(\omega)$ as its input. Compute the Fourier transform $Y(\omega)$ of $y(t)$.

(4) Describe the output signal $y(t)$ using the sine integral function $\mathrm{Si}(x)$:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}du.$$

(5) Sketch the graph of $y(t)$. Note that the shape of the sine integral function $\mathrm{Si}(x)$ is shown as Fig. in the next page. You may assume that $\omega_m t_w$ is sufficiently large.

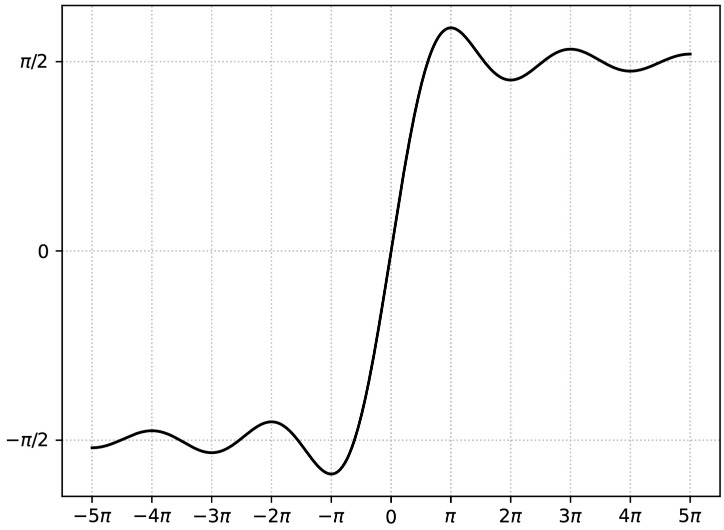

Fig.