2008 School Year

Graduate School
Entrance Examination Problem Booklet

Mathematics

Examination Time: 10:00 to 12:30

Instructions

1. Do not open this problem booklet until the start of the examination is announced.
2. If you find missing, misplaced, and/or unclearly printed pages in the problem booklet, ask the examiner.
3. Answer all of three problems appearing in this booklet, in Japanese or English.
4. You are given three answer sheets. You must use a separate answer sheet for each problem. You may continue to write your answer on the back of the answer sheet if you cannot conclude it on the front.
5. Fill the designated blanks at the top of each answer sheet with your examinee's number and the problem number you are to answer.
6. The blank pages are provided for rough work. Do not detach them from this problem booklet.
7. An answer sheet is regarded as invalid if you write marks and/or symbols and/or words unrelated to the answer on it.
8. Do not take the answer sheets and the problem booklet out of the examination room.

Examinee's number	No.

Fill this box with your examinee's number.

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Problem 1

Let A be the following matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Answer the following questions.

(1) Obtain A^2, A^3 and A^4.

(2) By generalizing the results of the previous question, obtain A^{3n}, A^{3n+1} and A^{3n+2} for nonnegative integers n.

(3) What is the characteristic polynomial of A?

(4) What is the minimal polynomial of A? Briefly explain the reason. (The minimal polynomial of A is the one with the smallest degree among the polynomials $m(x)$ such that $m(A) = O$ and the term of the largest degree has the coefficient 1.)

(5) Obtain the Jordan normal form J of A, and a regular matrix P such that $PJ = AP$.

Problem 2

Answer the following questions when throwing a dice repeatedly. Probability of each spot of the dice is $\frac{1}{6}$.

(1) Show the probability of casting first 1 at the n th throw.

(2) Let denote an event to cast the dice until first 1 as Event A. Show the expectation of number of throws for Event A.

(3) Show the variance of number of throws for Event A.

(4) Let denote an event to cast the dice until second 1 as Event B. Show the expectation and variance of number of throws for Event B by considering Event B as two independent Event A's.

(5) Show the expectation and variance of number of throws for Event B without assuming Event B as two Event A's. Confirm that the results coincide with those obtained in (4).

Problem 3

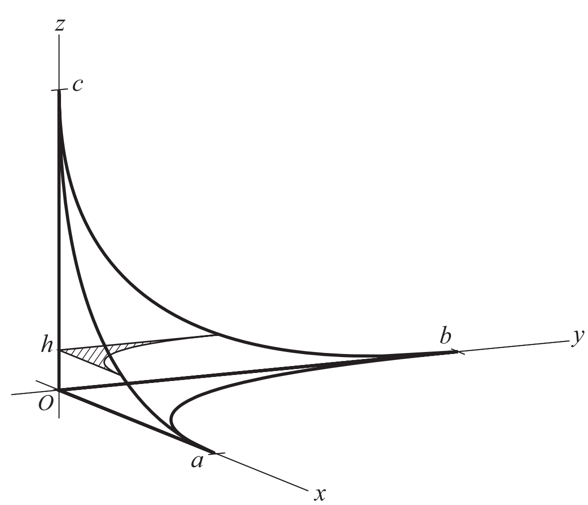
A surface Q is represented by the following equation:

$$\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{z}{c}} = 1 \qquad\qquad (a > 0,\ b > 0,\ c > 0)$$

Answer the following questions.

(1) Consider a region V that is surrounded by the surface Q and three planes $x = 0$, $y = 0$ and $z = 0$. Let $S(h)$ be the area of cross section of V cut by a plane $z = h$ that is perpendicular to the z-axis (see the figure). Calculate $S(h)$ and the volume of V.

(2) There is a plane P that contains the intersection points of the surface Q and each coordinate axis. Consider the tangent plane to the surface Q, which is parallel to the plane P. Give the equation of the tangent plane, and the position of the tangent point.



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