Master Course Entrance Examination Problem Booklet

Information Physics and Computing

19th (Tuesday) August 2014 10:00~13:00

This booklet is the English version of the entrance examination problem booklet in Japanese for your assistance. Note that the most accurate expression of the problem is in the Japanese version.

Answer three problems out of the six problems.

Note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to fill the examinee's number and the problem number in the designated place at the top of each answer sheet. Do never put your name.

(5) Do not separate the draft papers from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that the problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions to the original ones.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	No.

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with numbers of three problems you selected.

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Problem 1

Given a real-valued time signal $x(t)$, we define the Fourier transform $X(\omega)$ of $x(t)$ as

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \mathrm{d}t,$$

where ω represents an angular frequency and j is the imaginary unit. Also, we define the inverse Fourier transform as

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \mathrm{d}\omega.$$

Answer the following questions.

(1) Given the autocorrelation $r(p)$ of $x(t)$ in Eq. (1), show that $|r(p)| \leq r(0)$ for an arbitrary real number p. Next, prove the Wiener-Khinchin theorem by expressing the Fourier transform $R(\omega)$ of $r(p)$ with a function of $X(\omega)$. Also, show that $R(\omega)$ is a real-valued function but does not necessarily take the maximum at $\omega = 0$.

$$r(p) = \int_{-\infty}^{\infty} x(t) x(t+p) \mathrm{d}t \tag{1}$$

(2) Given $x(t)$ in Eq. (2), find the Fourier transform $R(\omega)$ of the autocorrelation of $x(t)$:

$$x(t) = \begin{cases} \sqrt{\frac{1}{2a}} & , \quad -a \leq t \leq a, \\ 0 & , \quad \text{otherwise}, \end{cases} \tag{2}$$

where a is a positive constant. Next, prove that the angular frequencies that give minima of $R(\omega)$ are periodic except for the region $-\pi/a < \omega < \pi/a$, and show the period in terms of a. Also, explain whether the angular frequencies that give maxima of $R(\omega)$ are periodic or not.

(3) We observed the signal $\hat{x}(t)$ in Eq. (3) via a sensor:

$$\hat{x}(t) = \begin{cases} \sqrt{\frac{1}{2a}} e^{-(t+a)^2/b} & , \quad t < -a, \\ \sqrt{\frac{1}{2a}} & , \quad -a \leq t \leq a, \\ \sqrt{\frac{1}{2a}} e^{-(t-a)^2/b} & , \quad t > a, \end{cases} \tag{3}$$

where a and b are positive constants. Suppose that the filter with the following frequency response $F(\omega)$ is applied to $\hat{x}(t)$:

$$F(\omega) = c_1 + jc_2\omega, \tag{4}$$

where c_1 and c_2 are real constants. Then, find the filtered output signal $y(t)$. Next, explain what kind of information the above-mentioned filter can detect if $c_1 = 0$ and $b \to 0$.

(4) Suppose that the energy of the signal is normalized as $\int_{-\infty}^{\infty} x(t)^2 \mathrm{d}t = 1$. Under the condition, we define $\Delta_{t\omega}$ in Eq. (5) for evaluating "degree of localization" in the time-frequency regions:

$$\Delta_{t\omega} = \int_{-\infty}^{\infty} t^2 x(t)^2 \mathrm{d}t \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 |X(\omega)|^2 \mathrm{d}\omega. \tag{5}$$

Find $\Delta_{t\omega}$ in the case of Question (2) ($x(t)$ is given by Eq. (2)) and show that the value is not finite.

(5) Suppose the case that $\Delta_{t\omega}$ is finite. Find $x(t)$ that minimizes $\Delta_{t\omega}$ for obtaining the most localized function in the time-frequency regions according to the following steps.

(a) Represent $\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 |X(\omega)|^2 \mathrm{d}\omega$ in terms of the derivative $x'(t)$ of $x(t)$.

(b) Substituting the result of (a) into Eq. (5), derive an inequality that gives the lower bound of $\Delta_{t\omega}$.

(c) Find $x(t)$ that minimizes $\Delta_{t\omega}$.

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Problem 2

Figures 1, 2, and 3 show the operational amplifier circuits for an active filter, a sine-wave oscillator, and a load circuit of the oscillator, respectively. Answer the following questions. All the operational amplifiers can be assumed to be ideal.

(1) For Circuit 1 of Fig. 1, obtain a transfer function from V_{in} to V_{out}, and draw the frequency characteristics of the amplitude and phase as graphs.

(2) In Circuit 1 of Fig. 1, the resistance of R_3 was changed sufficiently slowly from 0 to a value much larger than those of R_1 and R_2. Explain how the DC gain and the cutoff frequency (corner frequency) of the active filter change.

(3) Circuit 2 of Fig. 2 is the circuit obtained by connecting the same circuits described in Fig. 1 in three stages as a loop. So that it works as a sine-wave oscillator, express the gain condition and the frequency condition using resistances R_1, R_2, variable resistance R_3, and capacitance C. Next, when $R_3 = 0$, the resistances of R_1 and R_2 were chosen so that the gain condition was satisfied, and then the resistance of R_3 was increased sufficiently slowly from 0 to a value much larger than those of R_1 and R_2. Explain how the oscillation changes.

(4) In the conditions when Circuit 2 of Fig. 2 is working as a sine-wave oscillator, express how the waveforms of output voltages $V_{\text{out1}}, V_{\text{out2}}$, and V_{out3} are mutually related. Next, the three outputs were connected to three resistors with identical resistance R_{L} of the load circuit shown in Fig. 3, which are connected in another side at a point. Obtain the voltage V_{n} of the connecting point, graphs of instantaneous dissipating power of the three load resistances R_{L}, and a graph of the sum of three instantaneous dissipating powers. Assume that the amplitude and phase of V_{out1} at time $t = 0$ are A and 0, respectively.

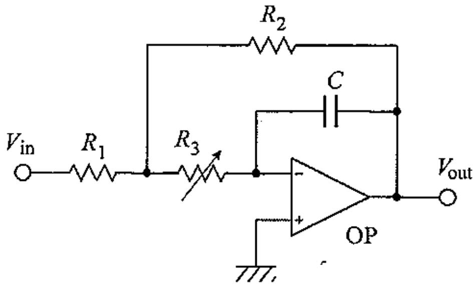
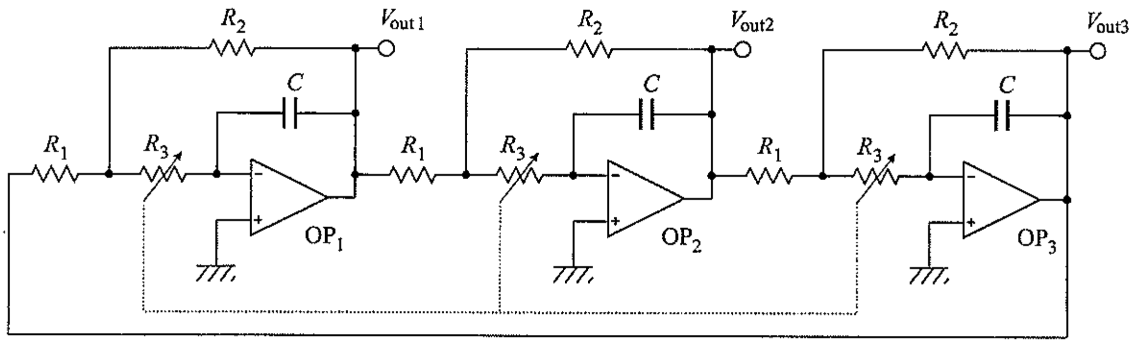


Fig. 1 Circuit 1



(The dotted lines indicate that the variable resistors change their resistances synchronously.)

Fig. 2 Circuit 2

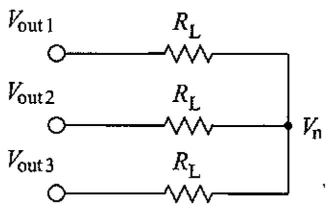


Fig. 3 Circuit 3

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Problem 3

Consider the control of an inversed pendulum in Fig. 1 and examine that the selection of the observed signals of controlled plants determines the difficulty of the feedback control design. As shown in Fig. 1, a ball of mass M [kg] is fixed at one end of a rod of length ℓ [m] and the other end is connected to an axis which can rotate in the x–y plane and is located on the ground. As in Fig. 1, θ [rad] is the angle of the rod to the vertically positive direction (the positive direction of y axis), $\kappa \frac{d\theta}{dt}$ [N·m] is the torque on the rotation axis caused by the friction and it is proportional to the rotation velocity, u [N·m] is the external torque on the rotation axis, and g [m·s^{-2}] is the gravitational acceleration of the gravity in the vertically negative direction (the negative direction of y axis). Suppose that $M = (2.45)^{-2}$ kg, $\ell = 2.45$ m, $\kappa = 3$ N·m·rad^{-1}·s, $g = 9.8$ m·s^{-2} and answer the following questions.

(1) (a) Suppose that the equation of motion of the inverted pendulum is given by

$$M\ell^2 \frac{d^2\theta}{dt^2} = M\ell g \sin\theta - \kappa \frac{d\theta}{dt} + u.$$

Let P_1 and P_2 denote an input-output system of (u, θ) and that of $(u, \frac{d\theta}{dt})$, respectively. By using linear approximation in $|\theta| \ll 1$, derive the transfer functions $P_1(s)$ and $P_2(s)$ of P_1 and P_2, respectively.

(b) Show the locations of the poles and the zeros of $P_1(s)$ and $P_2(s)$ on the complex plane, respectively.

(c) Show the outlines of the Nyquist diagrams of $P_1(s)$ and $P_2(s)$ with respect to the real axis. Also indicate the coordinates of their cross points with the real axis if they exist, respectively.

(2) Consider a feedback system in Fig. 2. Answer the following questions.

(a) Assume $C(s) = K$ (K is a positive real number). For each case that the transfer function of system P is $P(s) = P_1(s)$ or $P(s) = P_2(s)$ in Question (1)–(a), answer whether the feedback system shown in Fig. 2 is stabilizable or not by using the Nyquist stability criterion. When they are stabilizable, show the conditions on K for the stability of the feedback system.

(b) Assume $C(s) = \frac{K}{s+\alpha}$ (K and α are positive real numbers). For each case that the transfer function of system P is $P(s) = P_1(s)$ or $P(s) = P_2(s)$ in

Question (1)–(a), answer whether the feedback system shown in Fig. 2 is stabilizable or not by using the Routh–Hurwitz stability criterion. When they are stabilizable, show the conditions on K and α for the stability of the feedback system.

(3) Consider a general case of $P(s) = \frac{n(s)}{d(s)}$ and $C(s) = \frac{a(s)}{b(s)}$, where $n(s)$, $d(s)$, $a(s)$, and $b(s)$ are polynomials of s, and $P(s)$ and $C(s)$ are proper, respectively. Assume that a pair of $n(s)a(s)$ and $d(s)b(s)$ is coprime. Answer the following questions when the feedback system in Fig. 2 is asymptotically stable.

(a) Show that $n(s)a(s) + d(s)b(s)$ has the same sign in the region $0 \leq s$ on the real axis.

(b) Assume that the relative degree of $P(s)$ is 1 and $n(s)$ has a zero z ($< \infty$) in the region $0 \leq s$ on the real axis. Show that $d(s)b(s)$ has even number of zeros (including a case of no zeros) in the region $z < s < \infty$ on the real axis.

(4) For a case that the transfer function of system P is $P(s) = P_2(s)$ in Question (1)–(a) and $C(s) = \frac{K}{s+\alpha}$ (K and α are real numbers), answer the following questions.

(a) Show the condition on α for satisfying the condition answered in Question (3)–(b). Also explain its meaning from the viewpoint of control engineering.

(b) For a case that $\alpha = -2$, show the condition on K for the asymptotic stability of the feedback system in Fig. 2.

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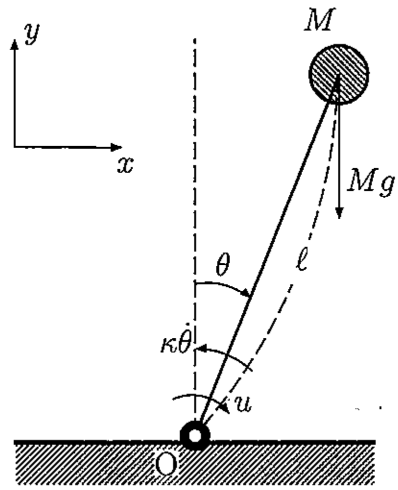

Fig. 1

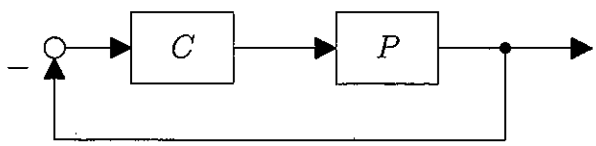

Fig. 2

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Problem 4

Answer the following questions concerning on binary multipliers.

(1) Show the truth table for a 1-bit full adder (abbreviated as FA hereafter). Let A and B be the inputs to the adder, CI be the carry-in from the lower bit, S be the sum, and CO be the carry-out to the upper bit. Next, express S and CO in a minimal sum-of-products form respectively.

First, we consider a multiplier of unsigned binary numbers.

(2) Figure 1 illustrates how $m_5m_4m_3m_2m_1m_0$, the result of multiplying two 3-bit binary numbers $x_2x_1x_0$ and $y_2y_1y_0$, is obtained. Here, p_{ij} is equal to $x_i \cdot y_j$, where " · " represents logical AND. We implement the calculation of Fig. 1. RCA_4 (abbreviation of ripple carry adder 4bit) has four FAs of Question (1) which are connected in sequential. CSA_5 (abbreviation of carry save adder 5bit) has five FAs of Question (1) and outputs the carry-outs of those FAs as they are. Under the assumption that all the required p_{ij} are generated in advance, the result of $m_5m_4m_3m_2m_1m_0$ can be obtained by using one CSA_5 and one RCA_4 as shown in Fig. 2.

(a) Complete the multiplier of Fig. 2. Indicate the connections between FAs explicitly.

(b) Among the paths from the input to the output in this multiplier, find the largest number of FAs which are passed through in a single path.

$$
\begin{array}{ccccccc}
 & & & & x_2 & x_1 & x_0 \\
 & & & \times & y_2 & y_1 & y_0 \\
\hline
 & & & & p_{20} & p_{10} & p_{00} \\
 & & & p_{21} & p_{11} & p_{01} & \\
+ & & p_{22} & p_{12} & p_{02} & & \\
\hline
 & m_5 & m_4 & m_3 & m_2 & m_1 & m_0
\end{array}
$$

Fig. 1

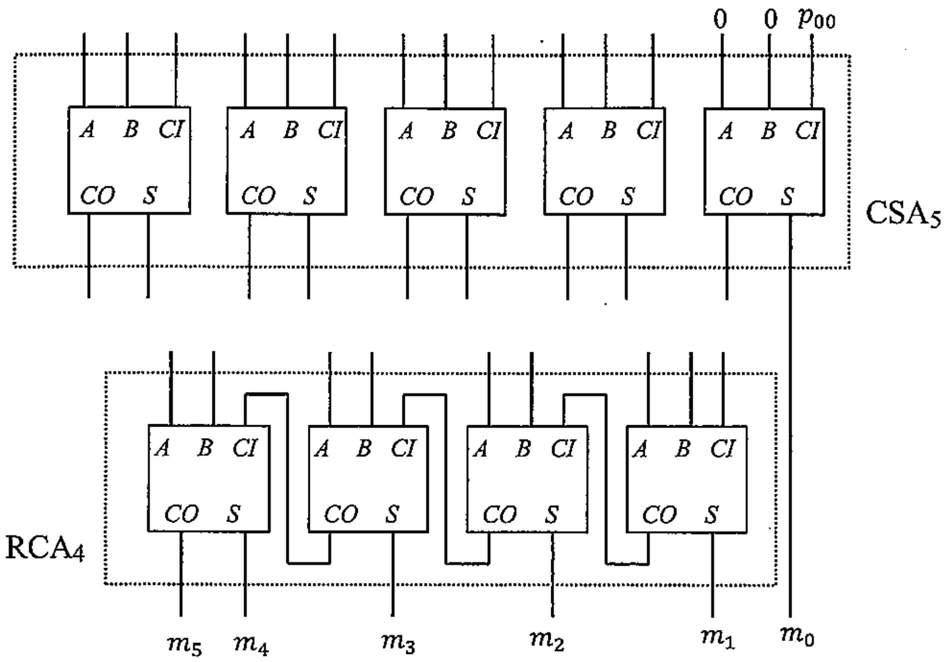

Fig. 2

(3) By extending the circuit of Question (2), we can implement a multiplier of two 6-bit binary numbers $x_5x_4x_3x_2x_1x_0$ and $y_5y_4y_3y_2y_1y_0$ by using one RCA and one or more CSAs. Any bit width can be chosen for each of the RCA and CSAs. Assume that all the required p_{ij} are generated in advance.

(a) Show the structure of the multiplier in which the number of FAs is as small as possible. In the answer, FAs in the RCA and CSAs are not necessarily shown explicitly, but indicate the bit width, the input, and the output of each of the RCA and CSAs, and the interconnections between them.

(b) Among the paths from the input to the output in this multiplier, find the largest number of FAs which are passed through in a single path.

Next, we consider a multiplier of two signed 3-bit binary numbers in two's complement notation. Since the result is in 6-bit width, the two signed 3-bit binary numbers should be extended into signed 6-bit binary numbers at first, and then multiplied. The result is given as the lower six bits of the multiplication.

(4) In two's complement notation, a signed 3-bit binary number $x_2x_1x_0$ is equal to a signed 6-bit binary number $x_2x_2x_2x_2x_1x_0$. Explain the reason.

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(5) The 6-bit result of $m_5m_4m_3m_2m_1m_0$ can be obtained through the calculation of Fig. 3. Some of values, however, are added multiple times in Fig. 3. Based on this observation, we implement a multiplier in which the number of FAs is as small as possible.

(a) Explain that $m_5m_4m_3m_2m_1m_0$ in Fig. 3 is the same as that of Fig. 4. Here, $\overline{p_{ij}}$ is the negation of p_{ij}.

(b) Even in Fig. 4, some of values are still added multiple times. Thus, another calculation which is simpler than Fig. 4 can be available. Based on this observation, show the structure of the multiplier in which the number of FAs is as small as possible. You can use one RCA and one or more CSAs. Any bit width can be chosen for each of the RCA and CSAs. Assume that all the required p_{ij} and $\overline{p_{ij}}$ are generated in advance. In the answer, FAs in the RCA and CSAs are not necessarily shown explicitly, but indicate the bit width, the input, and the output of each of the RCA and CSAs, and interconnections between them.

(c) Among the paths from the input to the output in this multiplier, find the largest number of FAs which are passed through in a single path.

$$
\begin{array}{rcccccc}
 & x_2 & x_2 & x_2 & x_2 & x_1 & x_0 \\
\times & y_2 & y_2 & y_2 & y_2 & y_1 & y_0 \\
\hline
 & p_{20} & p_{20} & p_{20} & p_{20} & p_{10} & p_{00} \\
 & p_{21} & p_{21} & p_{21} & p_{11} & p_{01} & \\
 & p_{22} & p_{22} & p_{12} & p_{02} & & \\
 & p_{22} & p_{12} & p_{02} & & & \\
 & p_{12} & p_{02} & & & & \\
+ & p_{02} & & & & & \\
\hline
 & m_5 & m_4 & m_3 & m_2 & m_1 & m_0
\end{array}
$$

Fig. 3

$$
\begin{array}{rcccccc}
 & x_2 & x_2 & x_2 & x_2 & x_1 & x_0 \\
\times & y_2 & y_2 & y_2 & y_2 & y_1 & y_0 \\
\hline
 & & & & \overline{p_{20}} & p_{10} & p_{00} \\
 & & & \overline{p_{21}} & p_{11} & p_{01} & \\
 & & \overline{p_{22}} & p_{12} & p_{02} & & \\
 & p_{22} & p_{12} & p_{02} & & & \\
 & p_{12} & p_{02} & & & & \\
 & p_{02} & & & & & \\
+ & 1 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & m_5 & m_4 & m_3 & m_2 & m_1 & m_0
\end{array}
$$

Fig. 4

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Problem 5

Consider a material particle of mass m that moves around the earth whose mass and radius are M and R, respectively. Setting the origin of the world coordinate system at the center of the earth O, taking the polar coordinate system in the plane including the origin, let (r, θ) be the coordinates of the material particle, and consider the motion of the particle in this plane. Suppose that only the universal gravitational force GmM/r^2 is working between the earth and the material particle, where G is the gravitational constant, and no other force is exerted to them. Here, assume that $M \gg m$ and the center of the gravity of the system that is composed of the material particle and the earth coincides with the center of the earth. Assume also that the effect of the air resistance can be ignored and the earth is a sphere and does not rotate. Answer the following questions.

(1) As shown in Fig. 1, let $\boldsymbol{r}$ be a position vector of the material particle where the origin is the center of the earth, $\boldsymbol{e}_r$ be the unit vector in the $\boldsymbol{r}$ (radius) direction, and $\boldsymbol{e}_\theta$ be the unit vector that is in the direction perpendicular to $\boldsymbol{e}_r$, respectively. Investigate the equation of motion with respect to the material particle in the polar coordinate system and show that

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = -\frac{GM}{r^2} \quad (1), \qquad 2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} + r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0. \quad (2)$$

Also show that $r^2 \frac{\mathrm{d}\theta}{\mathrm{d}t}$ becomes constant from the result. Hereafter we assume that this value is not zero and express it as h.

From the two differential equations in Question (1), it is demonstrated that (r, θ) of the material particle satisfy $r = l/(1 + \varepsilon \cos\theta)$, which is an equation of an ellipse in the polar coordinate system where $l = h^2/(GM)$. You can use these results without any proof, and answer the following questions.

(2) Show that the energy E of the material particle, which is the sum of the kinetic and potential energies, equals $-GmM/(2a)$, and is conserved without distinction of the value of θ when it is located at (r, θ). Here, "a" is the semi-major axis of the elliptical orbit, and the value of the potential energy according to the universal gravitation is defined such that it becomes zero where the distance from the center of the earth becomes infinity.

(3) As shown in Fig. 3, assume that the material particle is launched from the point P(R, θ_p) on the surface of the earth and only the universal gravitation of the earth works to the material particle after the launch of the material particle. Show that the length of the semi-major axis "a" of the elliptical orbit on which the launched material particle moves round only depends on the initial launching speed and does not depend on the launching angle. Calculate the value of a when the initial launching speed of the material particle is κv_β ($0 < \kappa < 1$). Here, v_β is a launching speed with which the launched material particle comes to move round the earth on the circular orbit with a radius R (the same value with the radius of the earth itself).

(4) In Question (3), assume that $\kappa = \sqrt{6}/3$. Calculate the longest value of the geodesic distance with which the launched material particle can reach from the launching point P to the point of fall Q. (Here, the "geodesic distance" means the distance which represents the length of the shortest curved line that connects the point P and the point Q on the surface of the earth).

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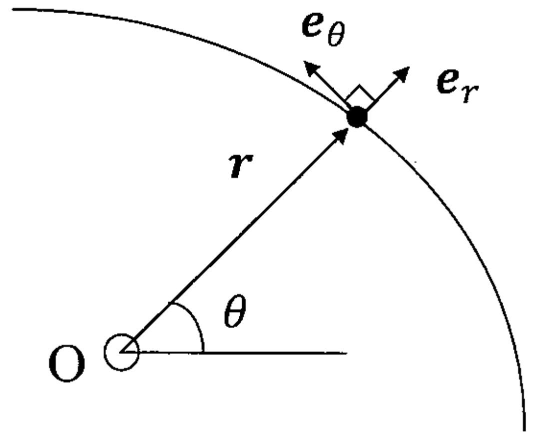

Fig. 1

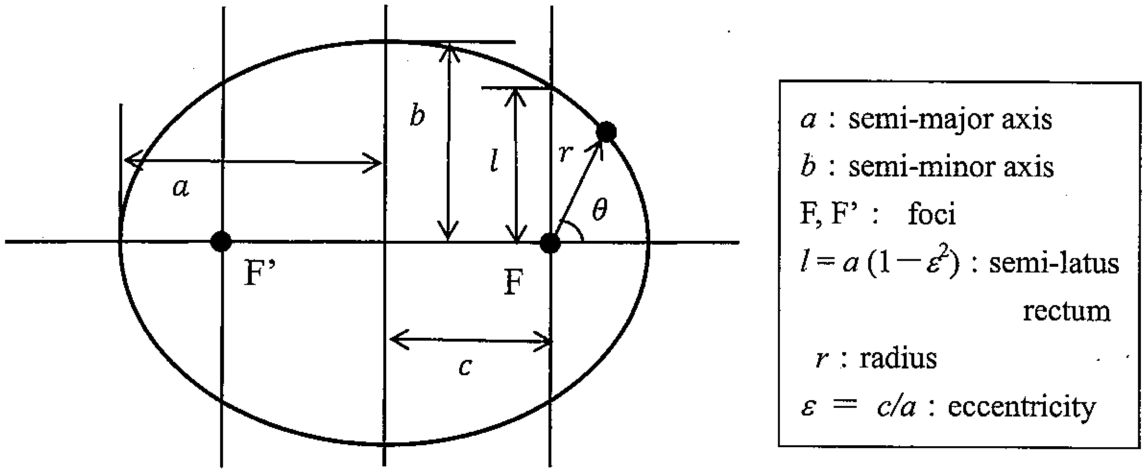

Fig. 2

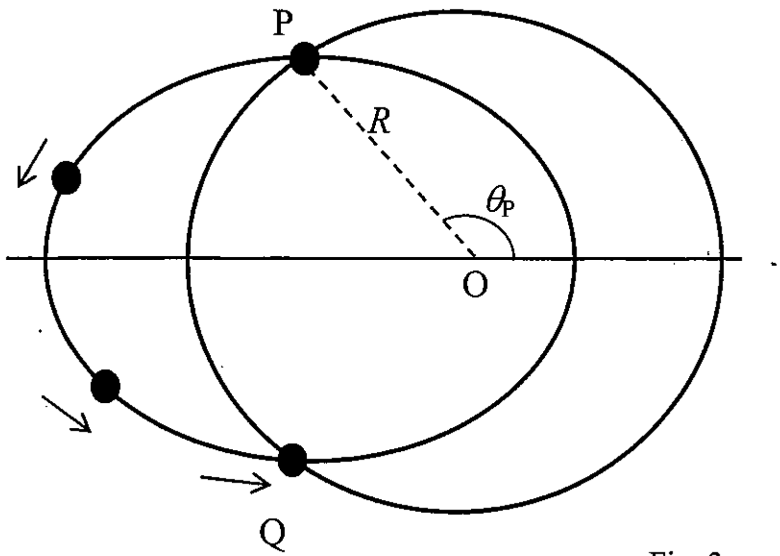

Fig. 3

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Problem 6

Answer the following questions.

(1) As shown in Fig. 1 with the x-z coordinates, monochromatic plane-wave light whose electric field is provided by Eq. (1) is incident perpendicularly to the interface P_1 (z=0) from medium 1 toward medium 2 which are dielectric materials. Here, the relative permittivities of medium 1 and medium 2 are ε_1 and ε_2 ($\varepsilon_1 < \varepsilon_2$), respectively, and the relative permeability μ of all the mediums in Problem 6 is 1.

$$E_0(t,x,z) = A\exp\left(j\omega\left(t-\frac{z}{v_1}\right)\right),\quad v_1 = \frac{c_0}{\sqrt{\varepsilon_1\mu}},\quad (z<0) \qquad (1)$$

Here, A is amplitude of the electric field, t is time, ω is angular-frequency of the light, c_0 is the velocity of light in vacuum, and j is the imaginary unit. Obtain the amplitude transmittance T_0 and the amplitude reflectance R_0 at the interface P_1. Suppose that there are no reflections at the interface $P_2(z=h)$ in this question. All conditions are homogeneous in the direction perpendicular to the x-z plane.

(2) In Question (1), consider multiple reflections at the interfaces P_1 and P_2, then the amplitude ratio T_{12} between the incident light at the interface P_1 and the emission light at the interface P_2 (z=h) is provided as Eq. (2). Derive Eq. (2).

$$T_{12} = \frac{1-\left(\frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}\right)^2}{1-\left(\frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}\right)^2\exp(j\delta)}\exp\left(j\frac{\delta}{2}\right),\quad \delta = -\frac{4\pi}{\lambda_0}h\sqrt{\varepsilon_2} \qquad (2)$$

Here, $\lambda_0 = 2\pi c_0/\omega$, and the interfaces P_1 and P_2 are flat and parallel to each other.

(3) In Question (2), obtain the condition on the thickness h which gives the maximum absolute value of T_{12}.

(4) As shown in Fig. 2, consider an optical thin membrane made of medium 2 in Question (1). The optical thin membrane consists of the light transmitting parts $S_1(-w < x < 0)$, S_2 $(0 \le x < w)$, and the light shielding parts $(x \le -w, x \ge w)$ whose transmittance is 0. The thickness of medium 2 at the light transmitting parts S_1 and S_2 are h_1 and h_2 $(h_1 < h_2)$, respectively. Place the optical thin membrane in medium 1, and then enter perpendicularly the monochromatic plane-wave light whose electric field is provided by Eq. (1) in Question (1) to the optical thin membrane. Obtain the condition that the absolute values of amplitude transmittances at both the light transmitting parts S_1 and S_2 become maximum, and the phase difference between the lights emitted from S_1 and S_2 at the interface P_5 becomes odd number multiple of π. Explain the characteristics of the energy distribution of the electric field of the emission light at the interface P_5. Note that the interfaces P_3 $(z = 0)$, P_4 $(z = h_2 - h_1)$, and P_5 $(z = h_2)$ of each light transmitting part are parallel to the x-axis. Suppose that the transmission of the electric field in the x-axis direction can be ignored. All conditions are homogeneous in the direction perpendicular to the x-z plane.

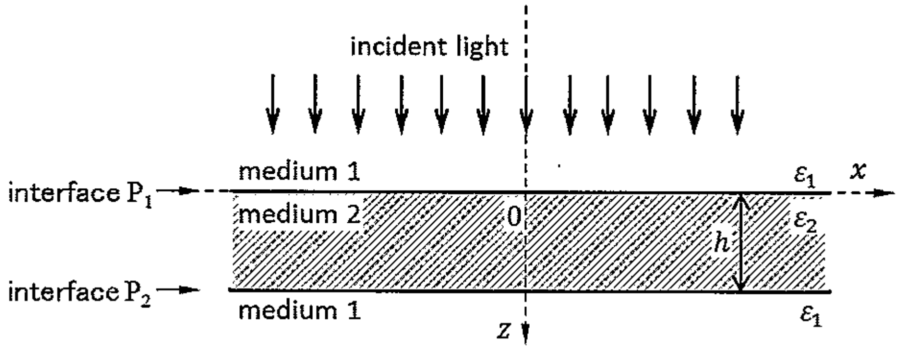

Fig. 1

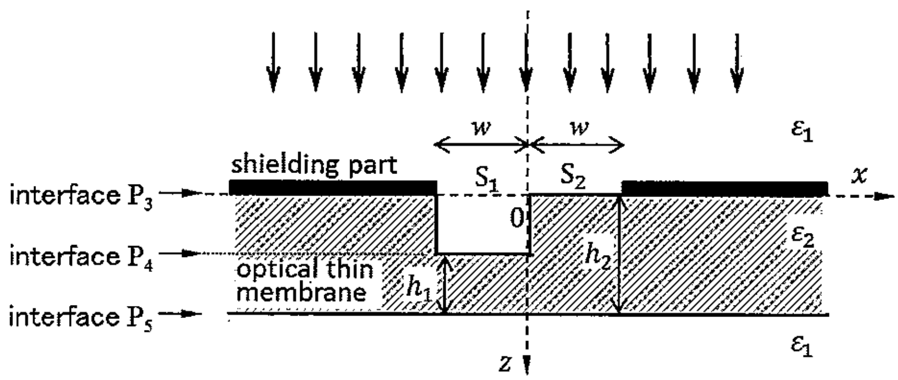

Fig. 2

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