Master Course Entrance Examination Problem Booklet

Information Physics and Computing

20th (Tuesday) August 2013　10:00~13:00

This booklet is the English version of the entrance examination problem booklet in Japanese for assistance. The most accurate expression of the problem is in the Japanese version.

Answer three problems out of the six problems.

Note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to write the examinee's number and the problem number in the designated place at the top of each answer sheet. Do never put your name.

(5) Do not separate the draft paper from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that the problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions to the original ones.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	No.

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with numbers of three problems you selected.

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Problem 1

For a discrete time signal $x[n]\,(n=-N,-N+1,\ldots,N; N \text{ is a positive integer})$, a transform is defined as

$$X[k]=\sum_{n=-N}^{N} x[n]\exp\left(-\mathrm{j}\frac{2\pi kn}{2N+1}\right) \quad (k=-N,-N+1,\ldots,N),$$

and it is called the F transform, where k is the discrete frequency and $\mathrm{j}=\sqrt{-1}$ is the imaginary unit. The inverse transform of the F transform is called the inverse F transform, and a pair of $x[n]$ and $X[k]$ is called an F transform pair when they have the F transform relationship. Answer the following questions regarding the F transform.

(1) Prove that the inverse F transform is expressed as

$$x[n]=\frac{1}{2N+1}\sum_{k=-N}^{N} X[k]\exp\left(\mathrm{j}\frac{2\pi kn}{2N+1}\right).$$

(2) Suppose that $x[n]$ and $X[k]$ are an F transform pair. Describe the characteristics of $X[k]$ when $x[n]$ is a real and even function. Also, describe the characteristics of $X[k]$ when $x[n]$ is a real and odd function.

Now, suppose that $x[n]$ and $X[k]$ are an F transform pair, $x[n]$ is a complex and analytic signal, and the DC component of the imaginary part of $x[n]$ is 0. Note that a signal is called analytic when the signal has no negative frequency components. Let the real and imaginary parts of $x[n]$ be $x_1[n]=\mathrm{Re}[x[n]]$ and $x_2[n]=\mathrm{Im}[x[n]]$, respectively. Let us find $x_2[n]$ from only the information on $x_1[n]$.

(3) Let the F transforms of $x_1[n]$ and $x_2[n]$ be $X_1[k]$ and $X_2[k]$, respectively. Express $X_2[k]$ by using $X_1[k]$ in the frequency domain.

(4) Express $x_2[n]$ by using $x_1[n]$ in the time domain.

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Problem 2

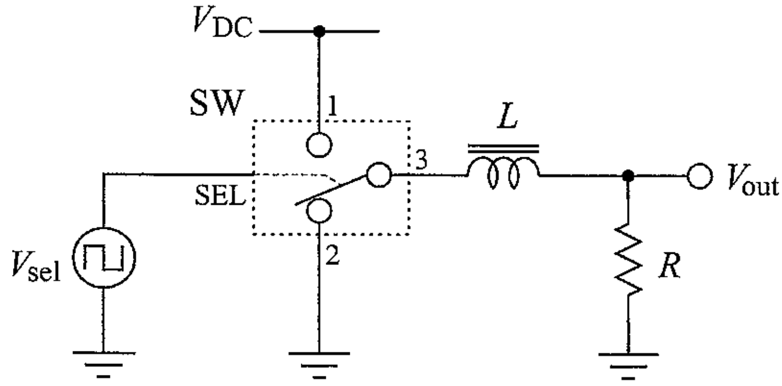
Figs. 1(a) and (b) show the circuits for converting the high-power-supply voltage $V_{\rm DC} > 0$ into a desired DC voltage and supplying it to the resistive load R with high efficiency. Answer the following questions. Here, assume that $V_{\rm CC} > 0$ is the power-supply voltage for the operational amplifier and digital circuits, $V_{\rm sel}$ is the digital source signal whose voltage is $V_{\rm CC}$ at logic 1, and 0 at logic 0, SW is the electronically controlled switch (switching time is ignorable) which connects terminal 3 to terminal 1 when the digital input SEL is logic 1, and connects terminal 3 to terminal 2 when SEL is logic 0, L is the inductance with a magnetic core whose magnetic saturation can be ignored, and OP is the operational amplifier which is ideal except that its output voltage is limited in the upper side at power supply voltage $V_{\rm CC}$ and in the lower side at 0.

(1) In the circuit 1 of Fig. 1(a), $V_{\rm sel}$ is 0 and the current through L is 0 during time $t < 0$. Then at $t = 0$, $V_{\rm sel}$ rises to 1, and begins continuously supplying the rectangle wave with period Δ_t in which the logic level in the former $a\Delta_t$ interval is 1 and the latter $(1-a)\Delta_t$ interval is 0 $(0 < a < 1)$. Obtain the equation that describes the temporal waveform of voltage $V_{\rm out}$ across R during $0 \le t < \Delta_t$, and draw it as a graph. Here, assume that $\Delta_t \ll L/R$.

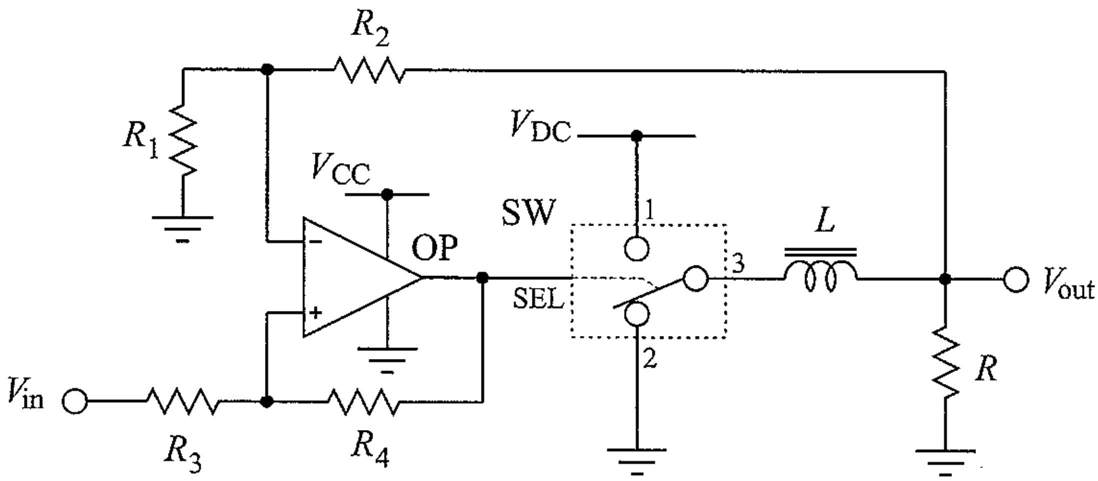
(2) Under the conditions of Question (1), let $V_{\rm out}$ at $t = n\Delta_t$ and $t = (n+a)\Delta_t$ be V_{2n} and V_{2n+1}, respectively (n is an integer greater than or equal to 1). Describe the above defined V_{2n+1} and V_{2n+2} using V_{2n}, and then, by solving the recursion equations, obtain the equations of V_{2n} and V_{2n+1} for arbitrary $n > 0$.

(3) In the circuit 2 of Fig. 1(b), a circuit block with the operational amplifier OP and the resistors R_3, R_4 is constructing the hysteresis comparator between voltage V_- of the inverted input terminal of OP and the input voltage $V_{\rm in}$ of the circuit 2. Explain the function and operation principle of the hysteresis comparator, and express the upper threshold voltage $V_{\rm H}$ and the lower threshold voltage $V_{\rm L}$ using $V_{\rm in}, V_{\rm CC}, R_3, R_4$. Here, assume that $0 < V_{\rm in} < V_{\rm CC}$.

(4) In the circuit 2 of Fig. 1(b), the input voltage $V_{\rm in}$ can vary in the range $0 < V_{\rm in} < V_{\rm CC}$, but its change is sufficiently slow. Explain that this circuit can operate as an oscillator when $V_{\rm in}, R_1, R_2$ are chosen appropriately with relating to $V_{\rm DC} > V_{\rm CC}$. Then, obtain the waveform and the oscillation frequency of the voltage $V_{\rm out}$ across R. Because $R_2 \gg R$, ignore the change of $V_{\rm out}$ caused by the current through R_2.



(a) circuit 1



(b) circuit 2

Fig. 1

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Problem 3

Answer the following questions on the control system in Fig. 1 which is composed of plant $P(s)$ and controller $C(s)$. Suppose that $r(t)$, $u(t)$, $y(t)$, and $\varepsilon(t)$ are scalar signals and the all internal states of the control system are zero at $t = 0$ in the following.

(1) Let $P(s)$ and $C(s)$ be $P(s) = \dfrac{1}{s^2 + s + 1}$ and $C(s) = K_1$ (K_1 is a positive constant), respectively. Show the Nyquist diagram of $P(s)$. Moreover, explain the possible locations of the poles of the stable closed loop system.

(2) Let $P(s)$ and $C(s)$ be proper rational functions with real coefficients.

(a) For a given $P(s)$, let us design $C(s)$ such that the transfer function $T(s)$ from $r(t)$ to $y(t)$ is stable and proper and no unstable pole-zero cancellations occur between $P(s)$ and $C(s)$. Explain realizable $T(s)$ under the above-mentioned conditions.

(b) Let $P(s)$ and $r(t)$ be $P(s) = \dfrac{1}{s+1}$ and the unit step signal $r(t) = 1$ $(0 \leq t)$, $r(t) = 0$ $(t < 0)$, respectively. Find $C(s)$ which is one of controllers satisfying $\lim_{t \to \infty} \varepsilon(t) = 0$.

(3) For $P(s) = \dfrac{1}{s+1}$, let us design $C(s)$ such that the transfer function $V(s)$ from $r(t)$ to $u(t)$ is $V(s) = K_2 \exp(-s)$ (K_2 is a constant).

(a) Show the block diagram of $C(s)$.

(b) Show the time response $\varepsilon(t)$ $(0 \leq t)$ for the unit step signal $r(t)$ ($r(t) = 1$ $(0 \leq t)$, $r(t) = 0$ $(t < 0)$).

(4) For $P(s) = \dfrac{1}{s+1}$, let us design $C(s)$ such that the transfer function $V(s)$ from $r(t)$ to $u(t)$ is $V(s) = K_3 + K_4 \exp(-s)$ (K_3 and K_4 are constants). Find K_3, K_4, and $C(s)$ which satisfy that the time response $\varepsilon(t)$ is $\varepsilon(t) = 0$ $(1 \leq t)$ for the unit step signal $r(t)$ ($r(t) = 1$ $(0 \leq t)$, $r(t) = 0$ $(t < 0)$).

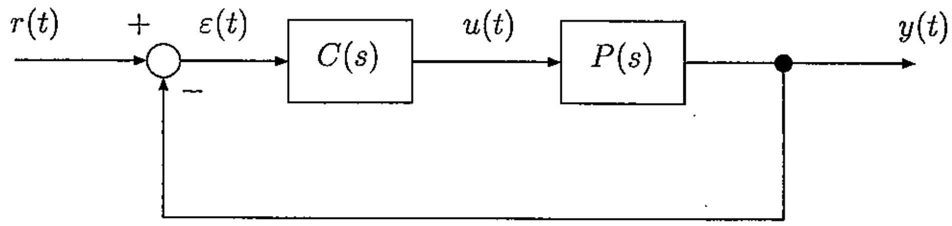

Fig. 1

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Problem 4

A Moore sequential machine is defined as a 5-tuple $M(Q, X, Z, \delta, \lambda)$ consisting of the following:

Q: a set of states

X: a set of input values

Z: a set of output values

$\delta \mid Q \times X \rightarrow Q$: a transition function

$\lambda \mid Q \rightarrow Z$: an output function

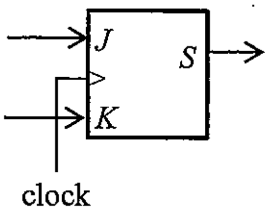
A Mealy sequential machine is also defined as a 5-tuple $M(Q, X, Z, \delta, \lambda)$, but its output function λ is given as $Q \times X \rightarrow Z$.

(1) Fig. 1 represents an edge-triggered JK flip-flop which accepts a clock signal. The output S changes at the rising edge of the clock signal. The new value of S (next S) is determined, as shown in Fig. 1, by the values of the inputs J and K, and the output S immediately before the rising edge of the clock signal. Fig. 2 is a sequential circuit including these two JK flip-flops. If a pair of the flip-flops outputs is considered as a state, this circuit implements a sequential machine in which the state transfers at the rising edge of the clock signal. Now, a Moore sequential machine $M_0(Q, X, Z, \delta, \lambda)$ of Fig. 3 is realized with a sequential circuit in Fig. 2. Express z, J_0, K_0, J_1, K_1 of Fig. 2 in a minimal sum-of-product form such that the values of $[S_0, S_1]$ are [0,0], [0,1], [1,0] at q_0, q_1, q_2, respectively.

(2) We derive a Mealy sequential machine M_1 equivalent to the Moore sequential machine M_0 of Fig. 3. $T(\omega\ ; M_1)$ represents an output sequence of M_1 to an input sequence ω. The length of $T(\omega\ ; M_1)$ is equal to that of ω. Meanwhile, $T(\omega\ ; M_0)$, an output sequence of M_0 to an input sequence ω, can be expressed as $\varepsilon T'(\omega\ ; M_0)$, because the beginning of the output sequence is ε ($\varepsilon \in Z$) that is the output depending only on the initial state. Here, the length of $T'(\omega\ ; M_0)$ is equal to that of ω. A Moore sequential machine M_0 is called to be equivalent to a Mealy sequential machine M_1 when $T'(\omega\ ; M_0) = T(\omega\ ; M_1)$ holds for any input sequence ω whose length is equal to or larger than 1.

(a) Show the output sequence when a sequence 00111 is input to the sequential machine of Fig. 3, that is, the length of the input sequence is five and this machine receives two 0s followed by three 1s. Note that the output sequence begins from 0 and its length is six because of the above-mentioned reason.

(b) Fig. 4 represents a Mealy sequential machine M_1 which has two states and is equivalent to the sequential machine M_0 of Fig. 3. The initial state of M_1 is q_0. Complete the tables of δ and λ.



S \ JK	00	01	11	10
0	0	0	1	1
1	1	0	0	1

next S

Fig. 1 JK flip-flop

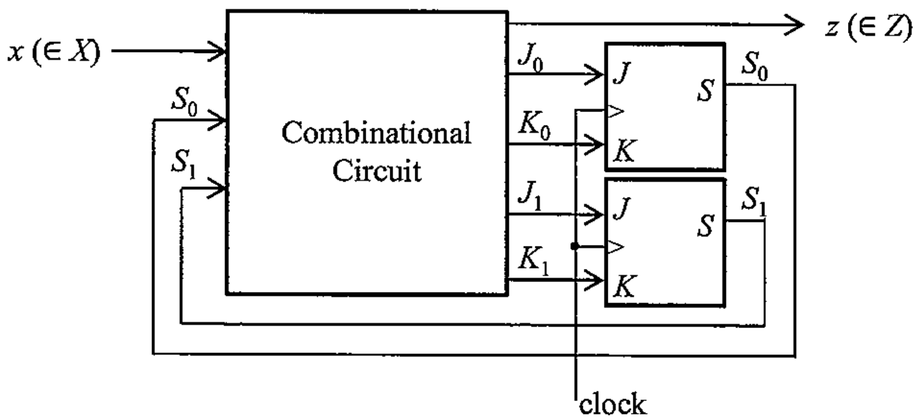


Fig. 2 Sequential Circuit

$X = \{0, 1\}$
$Q = \{q_0, q_1, q_2\}$
$Z = \{0, 1\}$
initial state : q_0

$\delta \mid Q \times X \rightarrow \text{next } Q$

Q \ X	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_2

next Q

$\lambda \mid Q \rightarrow Z$

Q	Z
q_0	0
q_1	0
q_2	1

Fig. 3 Moore Sequential Machine M_0

(Continued on the next page)

$X = \{0, 1\}$
$Q = \{q_0, q_1\}$
$Z = \{0, 1\}$
initial state : q_0

$\delta \mid Q \times X \rightarrow$ next Q

Q \ X	0	1
q_0		
q_1		

next Q

$\lambda \mid Q \times X \rightarrow Z$

Q \ X	0	1
q_0		
q_1		

Z

Fig. 4 Mealy Sequential Machine M_1

(3) Fill in the [blank] such that the following procedure derives a Moore sequential machine $M'(Q', X, Z, \delta', \lambda')$ equivalent to an arbitrary Mealy sequential machine $M(Q, X, Z, \delta, \lambda)$.

- A set of states of M' is defined as $Q' = \{ [q, z] \mid q \in Q, z \in Z \}$.
- The initial state of M' is given by a pair of $[q_0, z]$, where q_0 is the initial state of M and z is an arbitrary element of Z.
- The output function λ' is defined as follows.
  For $\forall q \in Q,\ \forall z \in Z, \lambda'([q, z])$ is z.
- The transition function δ' is defined as follows.
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(4) Prove that, on Question (3), $M'(Q', X, Z, \delta', \lambda')$ is equivalent to $M(Q, X, Z, \delta, \lambda)$.

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Problem 5

As an initial condition, consider a ball a_0 that is at a standstill on a straight line xy on a horizontal plane and an infinite number of balls $a_1, a_2, a_3, \ldots, a_n, \ldots$ that are moving on the y side of the same straight line with initial velocities $v_1, v_2, v_3, \ldots, v_n, \ldots$, respectively, and separated as shown in Fig. 1. Here, the order of the locations of the balls is $a_0, a_1, a_2, a_3, \ldots, a_n, \ldots$ from the x side to the y side; let the masses of the balls $a_0, a_1, a_2, a_3, \ldots, a_n, \ldots$ be $m_0, m_1, m_2, m_3, \ldots, m_n, \ldots$, respectively and the value of $c_{n+1} = m_{n+1}/m_n$ (the ratio of the mass of the ball a_{n+1} to that of the ball a_n) be a constant c for all values of n (n is an integer greater than or equal to 0).

Here, assume that: 1) all the balls are equal in size; 2) the surfaces of the horizontal plane and all the balls are perfectly smooth and no frictional force is generated between them; 3) the impact between the balls is perfectly elastic, i.e. the coefficient of restitution equals 1; and 4) the air resistance against the balls is negligible.

Here, define the direction of the velocity from x to y as being positive.

Answer the following questions.

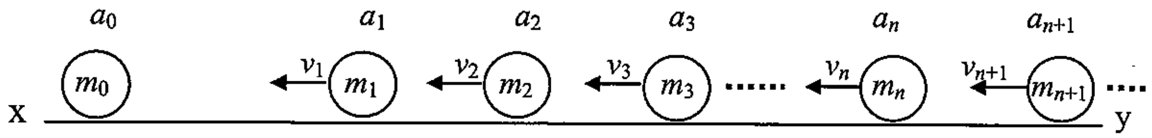

Fig. 1

(1) Suppose that the values of the initial velocities of the balls, $v_1, v_2, v_3, \ldots, v_n, \ldots$, are given by the following equation: $v_n = -v$ (here, n is a positive integer, and $v > 0$), and consider a chain collision that is generated as follows. First, the ball a_1 comes into collision with ball a_0, changes its velocity and in the same way comes into collision with the following ball a_2, and then the ball a_2 impacted by ball a_1 changes its velocity and comes into collision with the next ball a_3, ..., and so on. Thus make the balls generate this chain collision.
Calculate the velocities of balls a_0 and a_1 just after they collide (the first collision of the chain collision) using c and v, and show that no ball (except for a_0) will ever come into collision again with the balls after it first comes into collision with the adjacent ones.

(2) In Question (1), in terms of v, c and n, express the velocity of a_{n-1} and that of a_n just after the ball a_n comes into collision with the former ball a_{n-1} (n is a positive integer).
Now, let c be 1/2. Find the smallest number of n when the velocity of the ball a_n just after it comes into collision with the former ball a_{n-1} becomes greater than $19v$.
Here, let $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$, respectively.

(3)

(a) If the values of the initial velocities of the balls, $v_1, v_2, v_3, \ldots, v_n, \ldots$, are given by the following equation: $v_n = -v + pv(n-1)$ (here, n is a positive integer, and p is a constant); show that all the balls (except for a_0) come into collision with the adjacent ones only once if p satisfies $0 < p < (1-c)/(1+c)$, and then calculate the value of the velocity of the ball a_n just after it comes into collision with the former ball a_{n-1}.

(b) If the values of the initial velocities of the balls, $v_1, v_2, v_3, \ldots, v_n, \ldots$, are given by the following equation: $v_n = -vq^{n-1}$ (here, n is a positive integer, and q is a constant satisfying $0 < q < 1$), describe the condition under which all balls come to collide with the adjacent ones only once and then calculate the value of the velocity of the ball a_n just after it comes into collision with the former ball a_{n-1}.

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(4) As an initial condition, suppose that two balls a_{01} (mass: m_{01}) and a_{02} (mass: m_{02}) which are connected to each other by a spring of modulus k and with unstretched length l are at a standstill on the straight line xy instead of a_0, and the values of the initial velocities of the balls, $v_1, v_2, v_3, \ldots, v_n, \ldots$, are given by the following equation: $v_n = -v$ (here, n is a positive integer, and $v > 0$) (see Fig. 2). Here, assume that: 1) these two balls (a_{01} and a_{02}) are equal in size to the other balls ($a_1, a_2, a_3, \ldots, a_n, \ldots$); 2) their surfaces are perfectly smooth and no frictional force is generated; 3) the air resistance against the balls is negligible; 4) the impact between the ball a_1 and the ball a_{02} is perfectly elastic; and 5) $m_{02}/m_{01} = 1$, $m_1/(m_{01} + m_{02}) = c$. Assume also that the impact between the ball a_1 and the ball a_{02} occur instantaneously, and the mass of the spring and the size of the balls are negligible. Describe the movements of the balls a_{01} and a_{02} after the ball a_1 comes to collide with the ball a_{02} first, and compare the results with the case in which a single ball a_0 is used, namely as in Questions (1) and (2). Discuss also whether or not the situation in which the ball a_1 comes to collide with the ball a_{02} may occur again.

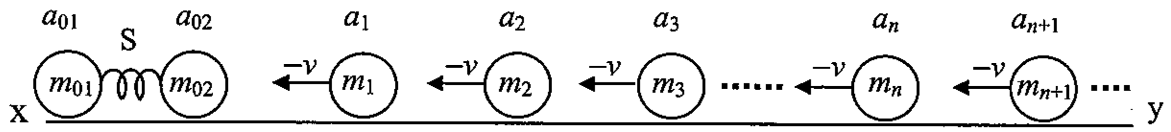

Fig. 2

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Problem 6

As shown in Fig. 1, through the cylindrical solenoid S_1 of length l_0, radius R, and number of turns N, a current I flows, where I is always kept constant. Set the central solenoid axis to be the z-axis. The current flows in counter-clockwise direction viewing from the positive z-direction. Throughout the problem, assume that the wire is infinitely thin and its mass is negligible. Also, the pitch angle α is negligible, and hence a single turn of the wire can be regarded to be circular perpendicular to the z-axis when the magnetic flux density is computed. The wire is mechanically fixed at $z = 0$ and $z = l_0$ with rigid insulators. The magnetic permeability of all the materials in this problem is μ_0. Answer the following questions.

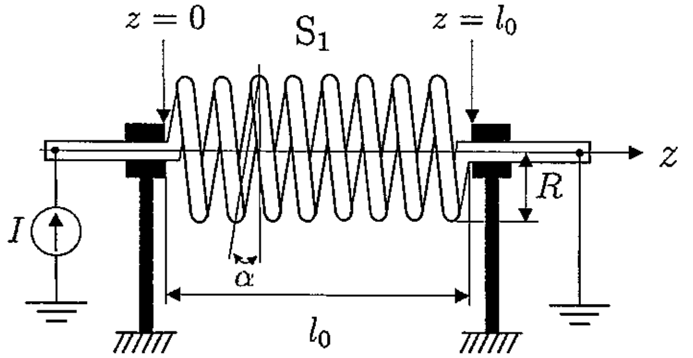

Fig. 1

(1) Show that the z-component of the magnetic flux density on the z-axis is given by

$$B(z) = \frac{\mu_0 NI}{2l_0}\left(\frac{l_0 - z}{\sqrt{(l_0 - z)^2 + R^2}} + \frac{z}{\sqrt{z^2 + R^2}}\right).$$

Here, the magnetic flux density generated by the current through the wire except S_1 is negligible. Note that the magnetic flux density generated by a current I through an infinitesimal element $\mathrm{d}\boldsymbol{s}$ at $\boldsymbol{r}$ viewing from $\mathrm{d}\boldsymbol{s}$ is given by the Biot-Savart law

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0}{4\pi} I\mathrm{d}\boldsymbol{s} \times \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}.$$

(2) Assume that the magnetic flux density in the cross section perpendicular to the z-axis can be regarded to be homogeneous. Show that the self-inductance, L,

of the solenoid S_1, which is defined by $L = \frac{\Phi}{I}$ where Φ is the magnetic flux passing through S_1, is given by

$$L = \mu_0 \pi R^2 \frac{N^2}{l_0}, \tag{1}$$

when $l_0 \gg R$.

Now, as shown in Fig. 2, the solenoid S_1, the point mass P with mass m which is a perfect conductor, and the solenoid S_2 with the same structure and winding direction as those of S_1 are connected in series with perfect conductors which are rigid bodies with negligible mass. Through them, a current I flows, where I is kept constant. Setting the central solenoid axis to be the z-axis, the end point of S_1 at $z = 0$ and that of S_2 at $z = a$ are mechanically fixed with rigid insulators. The point mass P whose rest point is at $z = \frac{a}{2}$ can have sufficiently small displacement only in the z-direction. Also, S_1 and S_2 can deform only in the z-direction, where their spring constants are k. In the following questions, the magnetic flux density generated by the current through the wire except S_1 and S_2 is negligible. Also, $R \ll l_0 \ll a$, and the magnetic flux density generated by one solenoid is homogeneous in the cross section perpendicular to the z-axis of another solenoid.

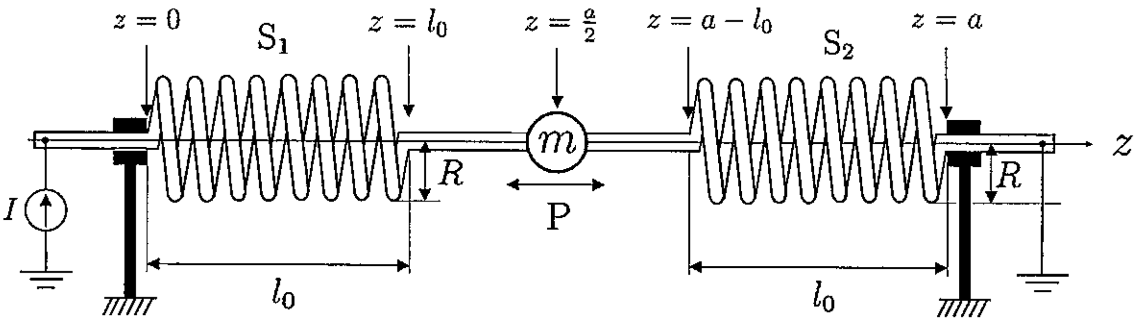

Fig. 2

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(3) Find the mutual inductance M of the solenoids S_1 and S_2, which is obtained as $M = \frac{\Phi_M}{I}$ where Φ_M is the magnetic flux passing through S_2 when a current I flows through S_1.

(4) An external driving force $F_0 \cos(\omega t)$ acts on the point mass P in the z-direction, where t is time and ω is an angular frequency. Accordingly, S_1 and S_2 deform in the z-direction, and the self inductances of S_1 and S_2 given in Eq. (1) when P is at $z = \frac{a}{2}$ change with time.

(a) Find the force exerted by S_1 on the point mass P when the displacement of P from $z = \frac{a}{2}$ in the z-direction is x where $0 < x \ll l_0$.

(b) Find the condition for resonance of the mechanical vibration of the point mass P in the z-direction.

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