Master Course Entrance Examination Problem Booklet

Information Physics and Computing

21st (Tuesday) August 2012 10:00~13:00

This booklet is the English version of the entrance examination problem booklet in Japanese for assistance. The most accurate expression of the problem is in the Japanese version.

Please answer three problems out of the six problems.

Please note:

(1) Do not open this booklet until the starting signal is given.

(2) You should notify the examiner if there are missing or incorrect pages in your booklet. No questions relating to the contents of the problems are acceptable in principle.

(3) Three answer sheets will be given. Use one sheet per a problem. You may use the back of the sheet if necessary.

(4) Do not forget to write the examinee's number and the problem number in the designated place at the top of all answer sheets. Do never put your name.

(5) Do not separate the draft paper from this booklet.

(6) Any answer sheet with marks or symbols unrelated to the answer will be invalid.

(7) In the case that the problem can be interpreted in several ways, you may answer the problem adding suitable definitions or conditions to the original ones.

(8) Do not take the answer sheets and this booklet out of the examination room.

Examinee's number	No.

Fill this box with your examinee's number.

Problem numbers you selected			

Fill these boxes with numbers of three problems you selected.

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Problem 1

(1) Answer the impulse response, $h(t)$, of an ideal band-pass filter (whose gain is 1 with no phase delay within the pass-bands and 0 elsewhere) with pass-bands $(-b,-a),(a,b)$, $0 < a < b$, in angular frequency.

(2) A rectangular wave $x(t), -\infty < t < \infty$ with fundamental angular frequency Ω is fed to the ideal band-pass filter described in Question (1) where $x(+0) = 1$, $x(-0) = -1$ and $x(-t) = -x(t)$. Answer the output, $y(t)$, of the filter.

(3) A time-discrete signal $y[n]$ (n is an integer, $-\infty < n < \infty$) is obtained by sampling $y(t)$ in Question (2) with angular frequency $\omega_s > 0$. Answer the discrete-time Fourier transform of $y[n]$:

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]\, e^{-jn\omega}$$

in respect to the normalized angular frequency, $-\pi \leq \omega < \pi$

(4) A white noise $w(t)$ with power σ^2 per unit band width is fed, as a common input, to the ideal band-pass filter mentioned in Question (1) and also to an ideal low-pass filter with a pass-band $(-c, c), c > 0$. The output signals are sampled at an angular frequency $\omega_s > 0$ similarly with in Question (3), and the sampled signals are denoted by $u[n]$ and $v[n]$, respectively. Answer the cross-correlation function between $u[n]$ and $v[n]$.

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Problem 2

Answer the following questions regarding the ideal operational amplifiers.

(1) (a) Obtain voltage gain v_{out}/v_{in} in Fig. 1. Draw the circuit diagram and obtain the voltage gain where R_1 tends to infinity and R_2 is equal to 0. Then, describe the role of the circuit in the meaning of electronic circuitry.

(b) Obtain voltage gain v_{out}/v_{in} in Fig. 2. Draw the circuit diagram and obtain the relationship between input current i_{in} and output voltage v_{out} where R_3 is equal to 0. Then, describe the role of the circuit in the meaning of electronic circuitry.

(2) Obtain output voltage v_{out} of the operational amplifier 2 (OP2) in Fig. 3.

(3) The circuit diagram of Fig. 4 is basically the same as that of Fig. 3 except for the modification that OP2 is removed and the terminal is grounded. Obtain current i_6 flowing along resistor R_6. By connecting the circuit diagram that you drew in Question (1)(a) and time-varying resistive device R_L shown in Fig. 5 to the circuit shown in Fig. 4, we want to drive R_L with constant voltage v_{in} and constant current i_6 that you obtained in this Question. Draw the circuit diagram to realize this driving scheme and describe the driving mechanism. Note that 'connecting' means the operations: attaching electric components on existing lines and/or inserting electric components in existing lines.

(4) When the electric power (voltages: $V_{CC}>0$ and $-V_{CC}$) to the operational amplifier (OP) in Fig. 6 was begun to supply at t=0, output voltage $v_{out}(t)$ began to oscillate with its maximum amplitude of $\pm V_{CC}$. Obtain the oscillation period of $v_{out}(t)$, and sketch the wave of $v_{out}(t)$ and that of voltage $v_m(t)$ at the inverting input terminal of OP. Note that capacitor C has no electric charge at t=0, $v_{out}(0)=V_{CC}$, and all of the diodes D1, D2, and D3 have the ideal characteristics. Then, describe the behavior of $v_{out}(t)$ after turning on SW.

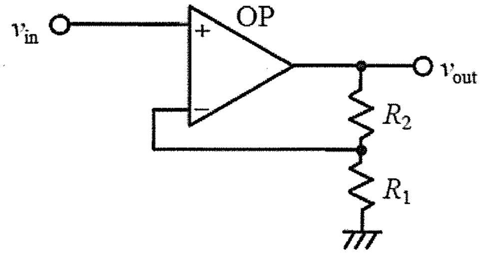

Fig. 1

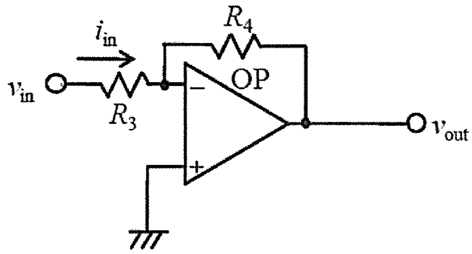

Fig. 2

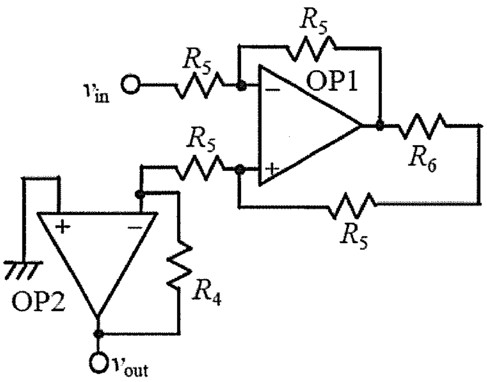

Fig. 3

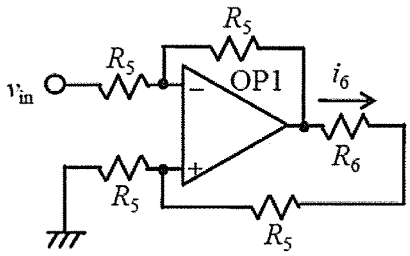

Fig. 4

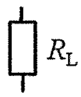

Fig. 5

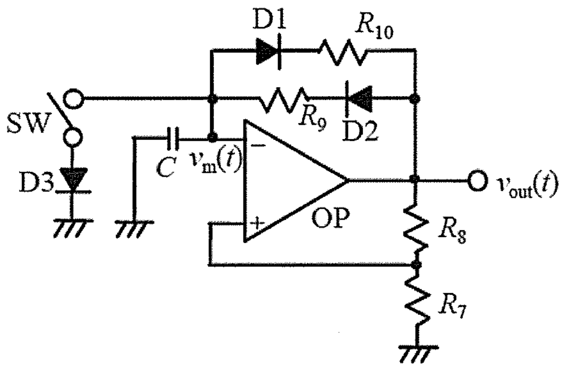

Fig. 6

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Problem 3

Consider a system shown as a block diagram in Fig. 1, where u and y denote the input and the output, respectively. Answer the following questions.

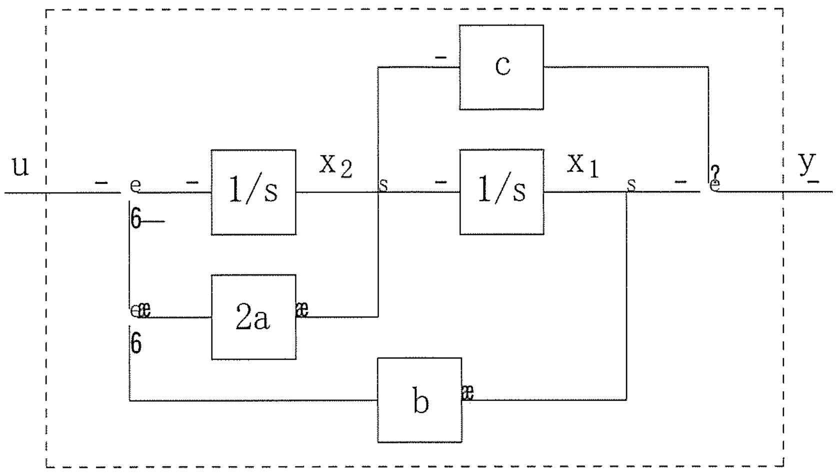

Fig. 1

(1) Derive a state-space realization of the system depicted in Fig. 1 and show its transfer function G(s).

(2) (a) Suppose $a = 11/2$, $b = 10$, and $c = 1/2$. Show the Bode diagram of the gain characteristic of G(s), or $|G(j\omega)|$, by approximating it with line segments, where $j = \sqrt{-1}$.

(b) Assume the initial state of the system depicted in Fig. 1 is zero. We observe that the output y(t) for a step-type input u(t) is given by

$$y(t) = 2e^{-3t} - 3e^{-t} + 1, \quad t > 0 .$$

Derive the parameters (a, b, c) and sign and magnitude of the step-type input which are consistent with the observation. Explain the reason if there exist no such parameters (a, b, c).

(3) Assume that $c = 2$ and $b > a^2$ for the system depicted in Fig. 1. Answer the following questions.

(a) Consider a feedback control system with $u(t) = -ky(t)$, $k > 0$ for G(s), where we further assume $a < 0$. Plot the root locus of the feedback control system and derive the stability condition.

(b) Derive the necessary and sufficient condition for the existence of an integral-type controller given by

$$K_I(s) = \frac{1}{Ts}, \quad T > 0$$

which stabilizes $G(s)$. Explain the reason if there exists no stabilizing integral-type controller for $G(s)$.

(4) Assume $c = 0$, or equivalently $y(t) = x_1(t)$, for the system depicted in Fig. 1. Answer the following questions.

(a) We want to obtain an estimate $\hat{x}_2(t)$ for $x_2(t)$ by using a first-order system represented by

$$\begin{aligned} \frac{d}{dt}z(t) &= f\,z(t) + gy(t) + hu(t) \\ \hat{x}_2(t) &= z(t) + \gamma y(t)\,. \end{aligned}$$

Determine the parameters (f, g, h, γ) so that

$$\lim_{t\to\infty} (\hat{x}_2(t) - x_2(t)) = 0$$

holds for any initial state variables $x_1(0)$, $x_2(0)$, and $z(0)$.

(b) Synthesize a controller which internally stabilizes the system of Fig. 1 by using the estimate $\hat{x}_2(t)$ for $x_2(t)$ constructed in Question (4)(a).

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Problem 4

For a Boolean function · ·· ·· ·· ·, we want to obtain a Boolean expression of · which satisfies the following Boolean equation,

· · · · · · · · · · · · · (*)

where · · · · · · · · are Boolean variables. Answer the following questions, where "·", "· ", and "·" represent the logical AND, the logical OR, and the logical NOT of ·, respectively.

(1) As a simple example, the following Boolean equation is given.

· · · · · · · · ·

First, obtain the solution · of this Boolean equation under a condition of · · · · · · · ·. Next, show an example of values of · · · · · · · · where any solution · does not exist for this Boolean equation.

(2) Any Boolean function · ·· ·· ·· · can be rewritten as follows.

· ·

By using this expression, prove that · ·· ·· ·· ·· · · ·· ·· ·· ·· · · is a necessary and sufficient condition that a solution · exists for the Boolean equation (*).

(3) Under the condition of Question (2), express the solution · of the Boolean equation (*) in a minimal sum-of-products form by using · ·· ·· ·· · and · ·· ·· ·· ·.

(4) Prove that a function · ·· ·· ·· · · · · · · · · · · · · · can be realized by connecting exclusive-OR gates and a combinational logic · ·· ·· · as shown in Fig.1, by using the result of Question (2). Then, express · ·· ·· · in a minimal sum-of-products form.

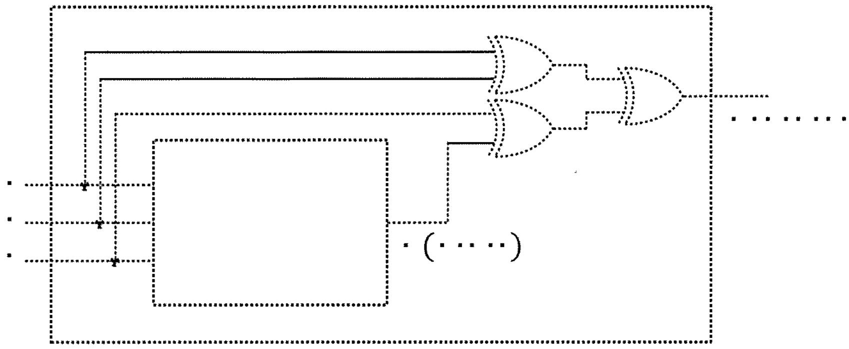

Fig. 1

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Problem 5

A cubic (regular hexahedron) particle is falling down uprightly without any rotational · motion in a reactant gas as shown in Fig. 1. Assume that $m(t)$[kg] is the mass of the cubic particle at time t[s]. The cubic particle grows in an isotropic manner under the growth rate $\mathrm{d}m(t)/\mathrm{d}t = bt$, where the growth rate coefficient b[kg/s^2] · · is constant. The particle shape is always cubic during the crystal growth. The initial mass of the cubic particle is m_0 [kg] and the initial velocity is 0 m/s at time t=0. This cubic particle has density · [kg/m^3], the gravitational acceleration is · [m/s^2], and the axis of coordinate is positive in the downward direction.

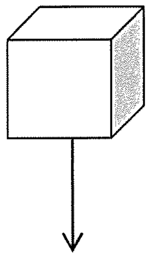

Fig. 1

(1) Obtain the falling velocity $v_1(t)$ [m/s], t· 0 of the cubic particle, where the growth rate coefficient is zero, b=0. Assume that the drag force from the reactant gas is proportional to the velocity. The proportionality coefficient of the drag force, c[Ns/m] · ·, is constant and does not depend on the volume and shape of the cubic particle.

(2) Assume that the gas is reactant with the cubic particle in the case of b>0. Obtain the falling velocity of the cubic particle, $v_2(t)$, t· 0. Note that the drag force to the cubic particle is negligible in this question for the sake of simplification.

(3) Consider the situation that the cubic particle is uprightly floating in the reactant gas. The bottom face of the cubic particle is uprightly irradiated with a parallel laser light from the lower direction. Assume that the number of photons per a second and per a unit area at time t is $n(t)$. The momentum of single photon is p [kgm/s]. Since all of photons are specularly reflected at the bottom face under the perfect elastic collision condition, no angular moment to the cubic particle occurs. The cubic particle is growing under the same condition described in Question (2).

(a) Obtain the force $F(t)$ [N], $t \cdot 0$ caused by the parallel laser irradiation.

(b) Obtain the number of photons per a second and per a unit area, $n_1(t)$, $t \cdot 0$, required for stationarily floating anytime.

(4) The velocity of the cubic particle reached v_p [m/s] at time $t=t_1$, $t_1 \cdot 0$. Obtain the number of photons per a second and per a unit area, $n_2(t)$, $t \cdot t_1$, required for constant falling velocity v_p after t_1. Assume that the drag force condition to the cubic particle is the same as Question (1). The cubic particle has the same crystal growth condition and the same initial condition as Question (2) and the parallel laser irradiation condition is the same as Question (3).

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Problem 6

As shown in Fig. 1, two straight line guide-rails (which are made of electrically conductive wires) are laid on a horizontal plane A so that they become parallel with a distance d, and an electrically conductive wire JK with mass m is put on these two guide-rails so that it goes perpendicularly across and short-circuits them. Here, the diameter of the conductive wire JK can be ignored. Assume that no friction acts between JK and the guide-rails, and that JK moves on those two guide-rails keeping perpendicularly across the guide-rails. Suppose one end of the above-mentioned guide-rails extends endlessly while maintaining the parallel state, and the other end is connected with an electric circuit shown in Fig. 1 using two switches at a sufficient distance from the position of JK. Suppose also that a uniform magnetic field with magnetic flux density B is present in the direction that is perpendicular to the horizontal plane A as shown in Fig. 1. Note that the two guide-rails and JK are rigid and that their shapes never change even if forces are applied to them and that the electrical resistances of those two guide-rails and the contact resistances between JK and the two guide-rails can be ignored. Note also that the magnetic fields that are generated by the electric current which flows through the two guide-rails and JK, and the inductance of the electric circuit that is constructed by the two guide-rails and JK can be ignored.

Answer the following questions.

(1) Calculate the electromotive force generated between the ends of the electrically conductive wire JK when it moves in a uniform magnetic field with magnetic flux density B along the guide-rails with a velocity of v. ($v>0$ when JK moves to the right in Fig. 1.) Calculate also the force that acts on the electrically conductive wire JK when the electric current i flows through JK from J to K in the uniform magnetic field with magnetic flux density B.

(2) First, connect switches 1 and 2 to the battery, V_0, and the electrical resistance, R, respectively, and after charging the capacitor, C, fully, alter switch 1 to the guide-rail side. Let the time t when switch 1 was altered to the guide-rail side be 0, and the position of the electrically conductive wire JK, x, be 0 when $t=0$. Obtain the position of JK, x, as a function of time t. Here, assume that the electrical resistance of the electrically conductive wire JK is 0 and suppose that JK stays without movement when $t=0$.

(3) In Question (2), after charging the capacitor fully, disconnect both switches once. Alter switch 2 so that it connects with the inductance coil, L, and then alter switch 1 so that it connects with the guide-rail. Obtain the electric current i which flows through JK from J to K and the position of JK, x, as a function of time t. Note that the electrical resistance of the electrically conductive wire JK is 0 and let the time when switch 1 is altered to the guide-rail side be 0 in the same manner as in Question (2).

(4) Next, in Question (3), assume that JK has a uniform resistance of r per a unit length. Find the condition with which the electric current i shows an oscillatory change with respect to time, calculate its natural frequency, and show the changes in the electric current through JK from J to K with respect to time briefly using a graph.

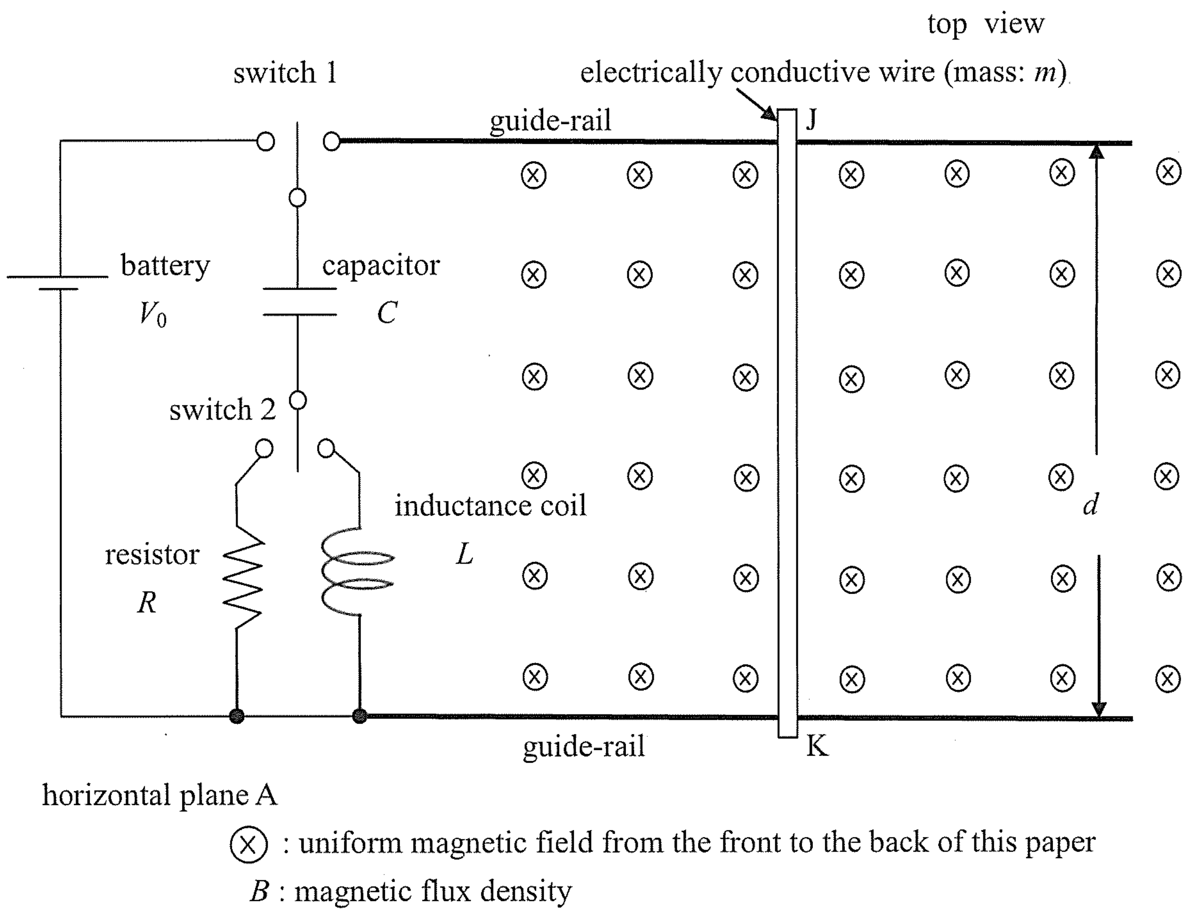

Fig. 1

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