Specialized Subjects

15:00-17:30, Monday, August 22, 2016

Instructions

1. Do not open this booklet before the examination begins.

2. This booklet contains five problems. The number of pages is five excluding this cover sheet and blank pages. If you find missing or badly printed pages, ask the proctor for exchange.

3. Answer three problems. You can select any three out of the five. Your answer to each problem should be written on a separate sheet. You may use the reverse side of the sheet if necessary.

4. Fill the top parts of your three answer sheets as instructed below. Before submitting your answer sheets, make sure that the top parts are correctly filled.

専門科目

写入問題
↑ Write the problem No.

受験番号
↑ Write the examinee No.

5. Submit all the three answer sheets with the examinee number and the problem number, even if your answer is blank.

6. Answer either in Japanese or English.

7. This booklet and the scratch paper must be returned at the end of the examination.

8. This English translation is supplemental and provided for convenience of applicants. The Japanese version is the formal one.
Problem 1

The figure shows a circuit consisting of an ideal operational amplifier connected to power supplies $V_S$ and $-V_S$, a capacitor $C$, resistors $R_1$, $R_2$, and $R_3$. When the circuit was powered up, the output voltage $V_{OUT}$ went toward positive and the input terminal voltage $V^-$ increased as well. Then $V^-$ started to decrease when $V^-$ reached $V_{TH}$. When $V^-$ reached $-V_{TH}$, $V^-$ turned upward and, eventually, $V^-$ underwent a periodic change.

Answer the following questions about the periodic operation of the circuit after a sufficiently long time has passed since the circuit was powered up. Note that $V_S > V_{TH} > 0$.

1. Calculate $V_{TH}$.

2. Calculate the transient response of the current $i$ flowing in the resistance $R_1$ while $V^-$ changes from $V_{TH}$ to $-V_{TH}$.

3. Calculate the repetition period of the circuit.

4. Calculate and sketch the time change of $V^-$ and $V_{OUT}$.
Problem 2

Let us consider a falling-edge-triggered JK flip-flop as shown in Fig. 1. This JK flip-flop is reset to \((Q, \overline{Q}) = (0, 1)\) when \(RST = 1\). Answer the following questions.

(1) Let us assume a 4-bit counter as shown in Fig. 2, and the input signals shown in Fig. 3 are given to the counter. Show a time chart of the outputs \(Q_0\)–\(Q_3\). Here, let us assume that all the \(J\) and \(K\) terminals are connected to 1, and the delay of each JK flip-flop, \(\tau\), cannot be neglected.

(2) Modify the circuit in Fig. 2 so that the outputs \(Q_0\)–\(Q_3\) change simultaneously, and show its schematic. (Such a circuit is called a parallel counter or a synchronous counter.)

(3) Let us design a parallel decimal counter. A decimal counter is a circuit that starts counting from \((Q_3Q_2Q_1Q_0) = (0000)\) up to \((Q_3Q_2Q_1Q_0) = (1001)\) and then returns to \((Q_3Q_2Q_1Q_0) = (0000)\) in the next state. Show a state transition diagram and a Karnaugh map of the counter.

(4) Show a schematic of a parallel decimal counter by using the results in (3).

(5) Modify the circuit in (2) to an up/down counter and show its schematic. An up/down counter is a counter whose operation mode can be changed either to count-up or count-down by a control signal \(UP/\overline{DOWN}\).

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**Fig. 1**

![Diagram of a JK flip-flop](image)

**Fig. 2**

![Diagram of a 4-bit counter](image)

**Fig. 3**

![Time chart](image)
Problem 3

You want to find users from an access log that extremely frequently access to a Web service. Unique user IDs are assigned to individual users, and the access log records IDs of the users that have accessed to the service in chronological order. Answer the following questions.

(1) You want to verify whether there exists a user that occupies the majority of the accesses in the access log, without creating a kind of a frequency histogram of all the users. You have therefore conceived the following algorithm.

   i. Prepare a list \( L \) that is initialized to an empty list \( \{ \} \).
   ii. Access each element in array \( A \) from the beginning, and perform either of the following operations for the \( i \)th element \( A[i] \) depending on the value of \( L \) at that time.
      ii-(a). (If \( L \) is empty) add \( A[i] \) to \( L \).
      ii-(b). (If \( L \) is not empty) if \( A[i] \) is included in \( L \), add \( A[i] \) to \( L \). Otherwise, remove one arbitrary element from \( L \).
   iii. output \( L \).

   Use this algorithm to process the following sequence of user IDs from the first element, and show the values of \( L \) in order after processing \( A[i] \) in ii.

   \[ 11, 10, 11, 11, 7, 11, 11, 3, 8 \]

(2) Prove that the number of varieties of user IDs in \( L \) is at most one in the algorithm in (1).

(3) If there exists a majority user \( u_{\text{MAJORITY}} \) in the access log, prove that \( u_{\text{MAJORITY}} \) is a unique element included in list \( L \) output by the algorithm in (1).

(4) In the algorithm in (1), when the size of array \( A \) is very large, the size of list \( L \) may cause an issue. Keeping this in mind, show a pseudo code that implements the algorithm in (1) using a function \texttt{read_log()} that reads a user ID one by one from the access log, while improving its space efficiency by not explicitly expressing \( L \) as a list.

Here, \texttt{read_log()} returns the first user ID in the access log at the first call and returns the following user IDs from the access log one by one for the following calls. \texttt{read_log()} returns \(-1\) when it reaches the end of the access log.
Problem 4

The Fourier transform of \( g^2(t) \) is defined as \( L(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\alpha)G(\omega - \alpha)d\alpha \). Here, \( G(\omega) \) is the Fourier transform of a signal \( g(t) \). Answer the following questions.

1. Briefly explain linear distortion of a transmission channel. When a pulse is input into a transmission channel with linear distortion, explain the effect on the pulse observed at the receiver.

2. Prove that the bandwidth of a signal \( x^2(t) \) is twice the bandwidth of \( x(t) \).

3. Assume that the output stage of the transmitter has a nonlinear characteristic for input \( x \) and output \( y \) as shown in the figure. Using an appropriate formula, explain the frequency modulated (FM) signal transmitted by this transmitter can be demodulated by a receiver without any effects from this nonlinearity. Here \( x \) and \( y \) are amplitude-limited as shown in the figure.

4. A modulated signal \( x(t) = m(t)\cos \omega_c t \) by an M-ary signal \( m(t) = \pm \frac{A}{2}, \pm \frac{2A}{2}, \ldots, \pm \frac{(M-1)A}{2} \) is transmitted through this transmitter. Here \( M \) is an even number. Briefly explain the problem observed in the demodulated signal at the receiver.

5. Assume that you have to multiplex signals over transmission channel with this nonlinear distortion. Comparing frequency-division multiplexing (FDM) with time-division multiplexing (TDM), choose the appropriate multiplexing method and show its rationale.
Problem 5

A model $M$ generated $a_n$ ($n = 0, \ldots, 15$) shown in the table below, working as an information source $S$. Answer the following questions. Use $\log_2 3 = 1.6$ and $\log_2 5 = 2.3$.

(1) Show the entropy of $S$ estimated from the table with two significant figures, assuming that each event of $S$ has been generated independently because $M$ is not clearly modeled now.

(2) Let us consider source coding for $S$. Design an encoding rule of $S$, whose average code length is less than 2.2 [bit].

Next, $M$ is now modeled as below.
Model $M$ generates $a_n$ according to $a_n = \alpha a_{n-1} + \beta a_{n-2} + b_n$ for $n \geq 2$. It is known that $b_n$ came from an information source $T$, whose entropy was 1.0 for generating the table. $\alpha$ and $\beta$ are integer constants, and satisfy $\alpha + \beta = 1$.

(3) Show an appropriate example pair of $\alpha$ and $\beta$. Also, describe the types of events and probabilities of occurrences for $T$.

(4) Calculate the code length needed for transferring the information of the table when $M$ is identified as above. Assume that $a_1$ and $a_0$ are given independently from $S$, and use the code designed in (2) for them.

(5) Let $I(X)$ be the entropy of an information source $X$. Generally, under a model, which has external inputs from multiple information sources $X_1, \ldots, X_n$ and provides an information source $Y$, $I(Y)$ is not always equal to $\sum_{i=1}^{n} I(X_i)$. Describe three scenarios that are responsible for such cases.

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