Written Exam

10:00 – 12:30, February 3, 2015

Entrance Examination (AY 2015)

Department of Computer Science
Graduate School of Information Science and Technology
The University of Tokyo

Notice:

(1) Do not open this problem booklet until the start of the examination is announced.

(2) Answer the following 4 problems. Use the designated answer sheet for each problem.

(3) Do not take the problem booklet or any answer sheet out of the examination room.

Write your examinee's number in the box below.

Examinee's number	No.

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Problem 1

(1) Suppose that A is a real symmetric matrix. Let $\boldsymbol{x}$ and λ be an eigenvector and the corresponding eigenvalue of A, respectively. Prove that λ is a real number.

(2) Suppose that A is a real symmetric matrix. Let $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ be eigenvectors of A; and λ_1 and λ_2 be the eigenvalues of A that correspond to $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, respectively. Furthermore, assume that $\lambda_1 \neq \lambda_2$.

Prove that $\boldsymbol{x}_1 \cdot \boldsymbol{x}_2 = 0$. Here $\boldsymbol{x}_1 \cdot \boldsymbol{x}_2$ denotes the inner product of the two vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$.

(3) Let A be a matrix

$$A = \begin{pmatrix} x & a_1 & b_1 & c_1 \\ y & a_2 & b_2 & c_2 \\ z & a_3 & b_3 & c_3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

where a_i, b_i and c_i ($i \in \{1, 2, 3\}$) are real numbers such that the two vectors $(b_1 - a_1, b_2 - a_2, b_3 - a_3)$ and $(c_1 - a_1, c_2 - a_2, c_3 - a_3)$ are linearly independent. Let us define a function $f\colon \mathbb{R}^3 \to \mathbb{R}$ by

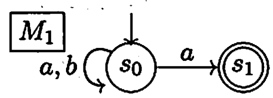
$$f(x, y, z) = \det(A) \ .$$

Prove that the equation $f(x, y, z) = 0$ determines a plane in the xyz space.

(4) In the setting of Question (3), give three points that lie on the plane determined by $f(x, y, z) = 0$.

Problem 2

Let us consider nondeterministic finite automata (NFA) and deterministic finite automata (DFA) over the alphabet $\Sigma = \{a, b\}$. For example, the NFA M_1 shown below



recognizes the language

$$L_1 \;=\; \{w \in \Sigma^* \mid \text{the last letter of } w \text{ is } a\} \;.$$

Here Σ^* denotes the set of words over Σ.

Answer the following questions.

(1) Present an NFA, with at most four states, that recognizes the language

$$L_3 \;=\; \bigl\{\, w \in \Sigma^* \bigm| |w| \geq 3 \text{ and the third last letter of } w \text{ is } a \,\bigr\} \;.$$

Here $|w|$ denotes the length of the word w.

(2) Present a DFA that recognizes the language L_3 in Question (1).

(3) A DFA that recognizes the language L_3 has no less than $2^3 = 8$ states. Prove this fact.

(4) Argue for the following statement:

> To recognize the same language, a DFA possibly needs the number of states that is exponentially bigger than an NFA does.

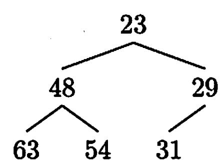
Problem 3

Consider the implementation of a priority queue using a binary heap data structure. Note that a binary heap satisfies the following properties.

- It forms a complete binary tree. All levels except for the deepest level are full, and the deepest level is filled from left to right.
- Each node has at most two child nodes, and the number stored in a parent node is equal or smaller than the numbers stored in its children.

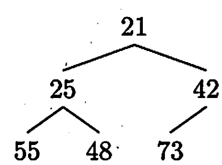
Answer the following questions.

(1) Suppose that you have the following binary heap.



Now you insert two elements, 21 and then 26, to the binary heap. Depict the step-by-step transformation of the binary heap.

(2) Suppose that you have the following binary heap.



Now you apply delete-min operations to the heap two times. Depict the step-by-step transformation of the binary heap.

(3) Estimate the number of nodes to be visited when inserting a node to a binary heap with n nodes. Give a brief explanation.

(4) Estimate the number of nodes to be visited when applying delete-min operation to a binary heap with n nodes. Give a brief explanation.

(5) Estimate the number of nodes to be visited when searching for the maximum element in a binary heap with n nodes. Give a brief explanation.

Problem 4

Consider the following task pool problem on a shared-memory system. Several worker processes concurrently execute the following program written in the C language:

```
while (1) {
  k = queue_head ++;
  if (k < N) {
    process(buffer[k]);
  } else {
    break;
  }
}
```

Among the processes, only the variable queue_head, the array buffer, and the constant N (the size of the array buffer) are shared. The value of queue_head is initially 0. The buffer is filled with data before execution, and each data must be processed exactly once.

Answer the following questions.

(1) The above processes may fail to work properly in an actual runtime environment, because mutual exclusion is not implemented. Show an example situation in which the above processes fail.

(2) Name at least two approaches to attain mutual exclusion.

(3) Choose one approach from your answers for Question (2). Explain its basic mechanism. Then explain how to solve the above task pool problem with it.

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