Written Exam

10:00 – 12:30, February 4, 2014

Entrance Examination (AY 2014)

Department of Computer Science
Graduate School of Information Science and Technology
The University of Tokyo

Notice:

(1) Do not open this problem booklet until the start of the examination is announced.

(2) Answer the following 4 problems. Use the designated answer sheet for each problem.

(3) Do not take the problem booklet or any answer sheet out of the examination room.

Write your examinee's number in the box below.

Examinee's number	No.

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Problem 1

Consider feeding behavior of a fish. Let θ ($0 < \theta < 1$) be the success probability of capturing a prey by a single trial, and assume that each trial is independent of the other trials. We consider an integer $x \geq 1$ such that the fish captures a prey for the first time at the x-th trial.

Let us first assume that θ is given as a constant number. Answer the following question.

(1) Find the probability mass function $P(x)$ of x. Calculate the expected value and the variance of x.

Let us now assume that θ has the probability density function given by

$$f(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\mathrm{B}(\alpha, \beta)} ,$$

where the beta function B is defined by

$$\mathrm{B}(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} .$$

Here $\alpha, \beta > 1$ are integers and the function Γ satisfies $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ and $\Gamma(1) = 1$.
Answer the following questions under this setting.

(2) Express the joint probability density function $P(x, \theta)$ using the beta function B.

(3) Express the probability mass function $P(x)$ of x, using the beta function B.

(4) Calculate the expected value, the variance, and the mode of θ. Here the mode of θ is the value of θ that maximizes $f(\theta)$.

Problem 2

Let $\Sigma = \{a, b, c\}$ be an alphabet; we shall consider nondeterministic finite automata (NFA) and deterministic finite automata (DFA) on Σ. The set of words on Σ is denoted by Σ^*.

Answer the following questions.

(1) Present an NFA, with four states, that recognizes the language $L_1 = \{waba \mid w \in \Sigma^*\} \cup \{wabb \mid w \in \Sigma^*\}$.

(2) Present a DFA that recognizes L_1 in Question (1).

Let M and M' be NFA on Σ, given concretely as below.

$$M = (Q, \Sigma, \delta, q_0, F) \qquad\qquad M' = (Q', \Sigma, \delta', q'_0, F')$$

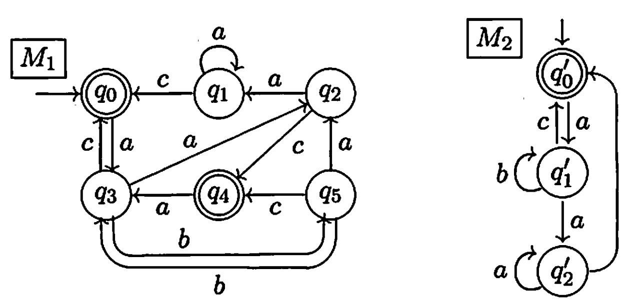
Here $\delta \subseteq Q \times \Sigma \times Q$ and $\delta' \subseteq Q' \times \Sigma \times Q'$ are transition relations; $q_0 \in Q$ and $q'_0 \in Q'$ are initial states; and $F \subseteq Q$ and $F' \subseteq Q'$ are the sets of accepting states, respectively.

A binary relation $R \subseteq Q \times Q'$ is called a *bisimulation* between M and M' if all the following conditions hold.

- $(q_0, q'_0) \in R$.
- For any states $q \in Q$ and $q' \in Q'$ such that $(q, q') \in R$,
  - $q \in F$ if and only if $q' \in F'$;
  - for any $l \in \Sigma$ and $s \in Q$, $(q, l, s) \in \delta$ implies that there exists $s' \in Q'$ such that $(q', l, s') \in \delta'$ and $(s, s') \in R$;
  - and conversely, for any $l \in \Sigma$ and $s' \in Q'$, $(q', l, s') \in \delta'$ implies that there exists $s \in Q$ such that $(q, l, s) \in \delta$ and $(s, s') \in R$.

Answer the following questions.

(3) Find a bisimulation R between the NFA M_1 and M_2 shown below. Here a double circle indicates that the state is accepting.



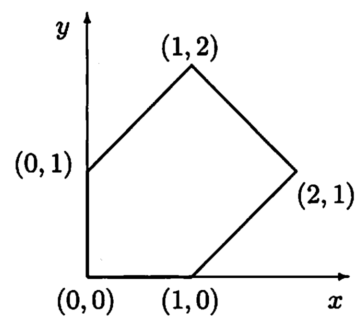
(4) Let $L(M)$ and $L(M')$ denote the languages recognized by NFA M and M', respectively. Show that, if there exists a bisimulation between M and M', then $L(M) = L(M')$.

Problem 3

Let us consider triangulation of an n-sided convex polygon $\{v_0, \ldots, v_{n-1}\}$. Here triangulation of a polygon means adding $n-3$ edges—each from one vertex of the polygon to another—to divide the polygon into $n-2$ disjoint triangles. The vertices of the resulting triangles must be those of the original polygon.

Answer the following questions.

(1) Show all the possible triangulations of the polygon shown below.



(2) Let $c[n]$ be the number of possible triangulations of an n-sided convex polygon. Show an equation that computes $c[n]$ when $c[i]$ are given for each $i < n$.

We now define the cost of a triangle as the sum of its edge lengths, and the cost of a triangulation as the sum of these costs. The distance between two vertices v_i, v_j shall be denoted by $|v_i v_j|$.

Answer the following questions.

(3) Show a triangulation of the polygon shown in Question (1) that minimizes the cost. Show also one that maximizes the cost.

(4) For $0 \le i < j \le n-1$, define $t[i, j]$ to be the minimal triangulation cost for the polygon $\{v_i, v_{i+1}, \ldots, v_j\}$. Show an equation that computes $t[i, j]$ for i, j such that $j - i = \ell$, when $t[i, j]$ for all i, j such that $j - i < \ell$ are already computed. Here assume that $t[i, j] = 0$ if $j - i < 2$.

(5) Show a pseudo-code that computes $t[0, n-1]$ by using the equation you have shown in Question (4). Draw a diagram that illustrates how the computation of the matrix $\big(t[i, j]\big)_{i,j}$ proceeds.

(6) Answer the computational time and space complexity of the algorithm you have shown in Question (5).

Problem 4

The `dwrite` function shown below is a system call which directly writes the data in the user memory area specified by `data`, whose size is `size` bytes, to the disk.

```
void dwrite(void *data, int size);
```

The `dwrite` system call takes 100 μsec in addition to the disk write time. The speed of the disk write is 100 MB/sec that does not depend on the write area nor size. For example, the `dwrite` takes 1.1 msec for writing 100 KB data.

In this problem, 1 KB and 1 MB are defined to be 10^3 and 10^6 bytes, respectively.

Answer the following questions.

(1) Show a formula calculating the effective write performance (in MB/sec), given the write size *ds* bytes. Calculate the effective write performance (MB/sec) in case of writing 10 KB data to the disk by `dwrite`.

(2) Write a code of the following `bwrite` library. The `bwrite` library writes the write data to the buffer kept in the library. The buffer size is 100 KB. In the `bwrite` library, the content of the buffer is written to the disk if the data size in the buffer reaches 100KB. You must use `dwrite` and the following memory copy function.

```
void memcpy(void *dst, void *src, int byte_size);
```

(3) Calculate the effective write performance (MB/sec) when `bwrite` is called 100,000 times, each with 1-byte data. Here suppose that the `memcpy` function has 500 MB/sec copy performance, and that the execution time of the `bwrite` library solely consists of that of `memcpy` and the `dwrite` system call.

(4) Describe a method to further improve the write performance of `bwrite` using threads.

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