2018 Summer Entrance Examination
Department of Creative Informatics
Graduate School of Information Science and Technology
The University of Tokyo

Programming

INSTRUCTIONS

1. Do not open this problem brochure until the signal to begin is given.

2. Write your examinee ID number below on this cover page.

3. An answer sheet and a draft sheet accompany this brochure. Write down your examinee ID number on these sheets.

4. You should have received a USB flash drive. Before the examination starts, copy the files from the USB flash drive to your PC. If you have some problems, consult the exam supervisor.

5. You may choose any programming language to answer.

6. You may consult only one printed manual of a programming language during the examination. You can use or copy any libraries or program fragments stored in your PC, but you should not connect your PC to the Internet.

7. By the end of the examination, make a directory/folder on your PC, whose name is the same as your examinee ID number, and copy your program files and related files into the directory/folder. Copy the directory/folder onto the USB flash drive that you received.

8. At the end of the examination, the USB flash drive, the answer sheet, and the draft sheet will be collected.

9. Keep your program execution environment open on your PC so that you can run your program as soon as possible during the oral examination in the afternoon.

10. Leave your PC and this brochure together in the room, and exit the room when the exam supervisor directs.

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Programming

Assume that matrix elements are non-negative integers and they are stored in main memory.

(1) When the algorithm below is used to multiply an \( m \times n \) matrix \( A \) and an \( n \times m \) matrix \( B \), how many times are these matrix elements in \( A \) and \( B \) read from the main memory? Write the total number of read operations on your answer sheet. Reading the same element twice is considered as two operations. Do not count accesses to matrix \( C \) or other variables.

```plaintext
var i = 0
while i < m begin
    var j = 0
    while j < m begin
        var d = 0
        var k = 0
        while k < n begin
            d = d + a_{i,k} * b_{k,j}
            k = k + 1
        end
        c_{i,j} = d
        j = j + 1
    end
    i = i + 1
end
```

(2) When an \( m \times n \) matrix is stored into a file, matrix elements are separated by a whitespace, rows are separated by a comma, and a period is written right after the last element in the last row. For example, the following \( 3 \times 4 \) matrix:

\[
\begin{pmatrix}
0 & 1 & 2 & 3 \\
4 & 5 & 6 & 7 \\
8 & 9 & 10 & 11
\end{pmatrix}
\]

is written into a file as follows:

0 1 2 3, 4 5 6 7, 8 9 10 11.

Answer the numbers of rows and columns in the matrix stored in file mat1.txt in the USB flash drive. Write the answer in your answer sheet. Ignore all letters following a period in the file.

(3) Compute the trace (the sum of the main diagonal elements) of the product of the matrices \( A \) and \( B \) stored in file mat1.txt and mat2.txt in the USB flash drive. Write the answer in your answer sheet.

(4) Once an element of a matrix is read from the main memory, it is saved in cache memory and, when the same element is required, the element is not read from the main memory as long as it is saved in the cache memory. The cache memory can hold at most \( s \) elements and
it is managed in the LRU (Least Recently Used) scheme. When the cache memory holds $s$
elements and a new element not included in the cache memory is required, the least recently
used element is discarded from the cache memory. Then the new element is read from the
main memory and saved in the cache memory.

Under this circumstance, how many times are the elements of $m \times n$ matrix $A$ and $n \times m$
matrix $B$ read from the main memory while multiplying them in the algorithm shown in (1)?
Write a program that computes the total number of read operations for given $m, n, p$, and $s$.

(5) Suppose that $m$ and $n$ share a common divider $p$. The algorithm for matrix multiplication
is changed as below so that the number of read operations from the main memory will decrease
when the cache memory mentioned in (4) is used. Fill in each of the blanks 1 to 6 with a
variable name. Write the answer in your answer sheet.

```
var u = 0
while u < m begin
    var v = 0
    while v < m begin
        var w = 0
        while w < n begin
            var i = u
            while i < 1 + 2 begin
                var j = v
                while j < 3 + 4 begin
                    var d = 0
                    var k = w
                    while k < 5 + 6 begin
                        d = d + a_{i,k} * b_{k,j}
                        k = k + 1
                    end
                    c_{i,j} = c_{i,j} + d
                    j = j + 1
                end
                i = i + 1
            end
        end
    end
end
```

(6) How many times are matrix elements in $A$ and $B$ read from the main memory during
matrix multiplication in the algorithm shown in (5) with the cache memory mentioned in (4)?
Write a program that computes the total number of read operations for given $m, n, p$, and $s$.

(7) When computing matrix multiplication as in (6), which common divider $p$ of $m$ and $n$
minimizes the total number of read operations from the main memory on matrix elements in
$A$ and $B$? Write a program that computes such $p$ (if several, the maximum $p$ among them)
for given $m, n, p$, and $s$. Moreover, write the result of the computation in your answer sheet for
$m = 200, n = 150$, and $s = 600$. 
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